Published by IOP Publishing for SISSA
Received: *September 30, 2008*
Accepted: *October 14, 2008*
Published: *October 31, 2008*

Flux interactions on D-branes and instantons

Marco Billò,[a] **Livia Ferro,**[ab] **Marialuisa Frau,**[a] **Francesco Fucito,**[c] **Alberto Lerda**[d] **and Jose F. Morales**[c]

[a] *Dipartimento di Fisica Teorica, Università di Torino and I.N.F.N. — sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy*
[b] *Laboratoire de Physique Théorique, École Normale Supérieure, 24, rue Lhomond, 75231 Paris Cedex 05, France*
[c] *I.N.F.N. — sezione di Roma II, Via della Ricerca Scientifica, I-00133 Roma, Italy*
[d] *Dipartimento di Scienze e Tecnologie Avanzate, Università del Piemonte Orientale and I.N.F.N. — Gruppo Collegato di Alessandria — sezione di Torino, Via V. Bellini 25/G, I-15100 Alessandria, Italy*
E-mail: `billo@to.infn.it`, `ferro@to.infn.it`, `frau@to.infn.it`, `lerda@to.infn.it`, `Francesco.Fucito@roma2.infn.it`, `Francisco.Morales@roma2.infn.it`

Abstract: We provide a direct world-sheet derivation of the couplings of NS-NS and R-R fluxes to various types of D-branes (including instantonic ones) by evaluating disk amplitudes among two open string vertex operators at a generic brane intersection and one closed string vertex representing the background fluxes. This world-sheet approach is in full agreement with the derivation of the flux couplings in the brane effective actions based on supergravity methods, but it is applicable also to more general brane configurations involving fields with twisted boundary conditions. As an application, we consider an orbifold compactification of Type IIB string theory with fractional D-branes preserving $\mathcal{N} = 1$ supersymmetry and study the flux-induced fermionic mass terms both on space-filling branes and on instantonic ones. Our results show the existence of a relation between the soft supersymmetry breaking and the lifting of some instanton fermionic zero-modes, which may lead to new types of non-perturbative couplings in brane-world models.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Flux compactifications.

JHEP10(2008)112

Contents

1. Introduction

A promising scenario for phenomenological applications of string theory and realistic model building is provided by four dimensional compactifications of Type II string theories preserving $\mathcal{N} = 1$ supersymmetry in the presence of intersecting or magnetized D-branes [1 – 3]. In these compactifications, gauge interactions similar to those of the supersymmetric extensions of the Standard Model can be engineered with space-filling D-branes that partially or totally wrap the internal six-dimensional space. By introducing several stacks of such D-branes one can realize adjoint gauge fields for various groups by means of the massless excitations of open strings that start and end on the same stack, while open strings that stretch between different stacks provide bi-fundamental matter fields. On the other hand, from the closed string point of view, (wrapped) D-branes are sources for various fields of Type II supergravity, which acquire a non-trivial profile in the bulk. Thus the effective actions of these brane-world models describe interactions of both open string (boundary) and closed string (bulk) degrees of freedom and have the generic structure of $\mathcal{N} = 1$ supergravity in four dimensions coupled to vector and chiral multiplets. Several important aspects of such effective actions have been intensively investigated over the years from various points of view [1 – 3].

One of the main ingredients of these string compactifications is the possibility of adding internal antisymmetric fluxes both in the Neveu-Schwarz-Neveu-Schwarz (NS-NS) and in the Ramond-Ramond (R-R) sector of the bulk theory [4 – 6]. These fluxes may bear important consequences on the low-energy effective action of the brane-worlds, such as moduli stabilization, supersymmetry breaking and, possibly, also the generation of non-perturbative superpotentials. At a perturbative level internal 3-form fluxes are encoded in a bulk superpotential [7, 8] from which F-terms for the various compactification moduli can be obtained using standard supergravity methods. These terms can also be interpreted as the θ^2 "auxiliary" components of the kinetic functions for the gauge theory defined on the space-filling branes, and thus are soft supersymmetry breaking terms for the brane-world effective action. These soft terms have been computed in refs. [9] - [16] and their effects, such as flux-induced masses for the gauginos and the gravitino, have been analyzed in various scenarios of flux compactifications relying on the structure of the bulk supergravity Lagrangian and on κ-symmetry considerations.

In addition to fluxes, another important issue to study is the non-perturbative sector of the effective actions coming from string theory compactifications [17, 18]. Only in the last few years, concrete computational techniques have been developed to analyze non-perturbative effects using systems of branes with different boundary conditions [19, 20]. These methods not only allow to reproduce [20 – 24] the known instanton calculus of (supersymmetric) field theories [25], but can also be generalized to more exotic configurations where field theory methods are not yet available [26 – 47]. The study of these exotic instanton configurations has led to interesting results in relation to moduli stabilization, (partial) supersymmetry breaking and even fermion masses and Yukawa couplings [26, 27, 34]. A delicate point about these stringy instantons concerns the presence of neutral anti-chiral fermionic zero-modes which completely decouple from all other instanton moduli, contrarily to what happens for the usual gauge theory instantons where they act as Lagrange multipliers for the fermionic ADHM constraints [20]. In order to get non-vanishing contributions to the effective action from such exotic instantons, it is therefore necessary to remove these anti-chiral zero modes [30, 31] or lift them by some mechanism [36]. The presence of internal background fluxes may allow for such a lifting and points to the existence of an intriguing interplay among soft supersymmetry breaking, moduli stabilization, instantons and more-generally non-perturbative effects in the low-energy theory which may lead to interesting developments and applications. Some preliminary results along these lines have recently appeared in ref. [46].

So far the consequences of the presence of internal NS-NS or R-R flux backgrounds onto the world-volume theory of space-filling or instantonic branes have been investigated relying entirely on space-time supergravity methods [48 – 53], rather than through a string world-sheet approach.[1] In this paper we fill this gap and derive the flux induced fermionic terms of the D-brane effective actions with an explicit conformal field theory calculation of scattering amplitudes among two open string vertex operators describing the fermionic excitations at a generic brane intersection and one closed string vertex operator describing the background flux. Our world-sheet approach is quite generic and allows to obtain the flux induced couplings in a unified way for a large variety of different cases: space-filling or instantonic branes, with or without magnetization, with twisted or untwisted boundary conditions. Indeed, the scattering amplitudes we compute are generic mixed disk amplitudes, *i.e.* mixed open/closed string amplitudes on disks with mixed boundary conditions, similar to the ones considered in refs. [55 – 58, 21].

Besides being interesting from a technical point of view, our approach not only reproduces correctly all known results but can be applied also to cases where the supergravity methods are less obvious, like for example to study how NS-NS or R-R fluxes couple to fields with twisted boundary conditions or how they modify the action which gives the measure of integration on the moduli space of instantons. Finding the flux-induced soft terms on instantonic branes of both ordinary and exotic type is a necessary step towards the investigations of the non-perturbative aspects of flux compactifications we have mentioned above.

In this paper, after discussing the general conformal field theory calculation of the flux couplings to boundary fermions in ten dimensions, we select a specific compactification of Type IIB string theory that leads to a brane-world theory with $\mathcal{N} = 1$ supersymmetry in four dimensions. In particular we consider $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of type IIB on $\mathcal{T}^6$ with fractional D-branes. In this compactification scheme string theory remains calculable and our explicit world-sheet approach is viable; moreover the existence of inequivalent types of fractional branes gives rise to a quiver structure allowing to engineer gauge theories with interesting contents, such as pure super Yang-Mills theory or SQCD. The non-perturbative side can then be explored by means of instantonic fractional branes which can be of both ordinary or exotic type.

More specifically, this paper is organized as follows: in section 2 we describe in detail the world-sheet derivation of the flux induced fermionic terms of the D-brane effective action from mixed open/closed string scattering amplitudes. The explicit results for various unmagnetized or magnetized branes as well as for instantonic branes are spelled out in section 3 in the case of untwisted open strings and in section 4 in some case of twisted open strings. The flux-induced fermionic couplings are further analyzed for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification which we briefly review in section 5. Later in section 5.2 we compare our world-sheet results for the flux couplings on fractional D3-branes with the effective supergravity approach to the soft supersymmetry breaking terms, finding perfect agreement. In section 6 we exploit the generality of our world-sheet based results to determine the soft terms of the action on the instanton moduli space, and finally in section 7 we summarize our results. Our conventions on spinors, on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold and on the flux couplings for wrapped fractional D9-branes are contained in the appendix.

[1]For some recent developments using world-sheet methods see ref. [54].

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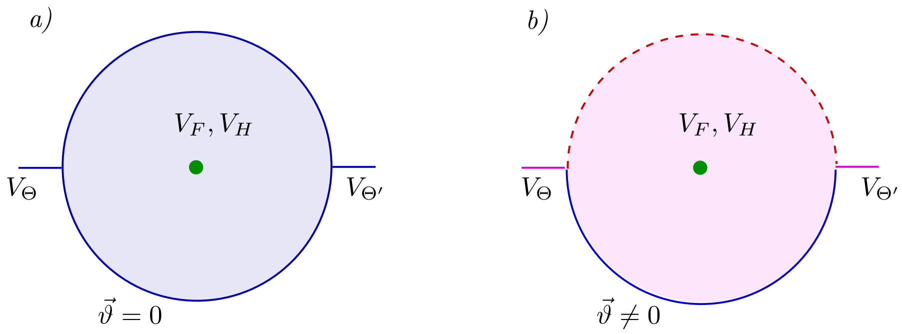

Figure 1: Quadratic coupling of untwisted, *a*), and twisted, *b*), open string states to closed string fluxes.

2. Flux interactions on D-branes from string diagrams

In this section, using world-sheet methods, we study the interactions between closed string background fluxes and massless open string excitations living on a generic D-brane intersection. We focus on fermionic terms (like for example mass terms for gauginos), but our conformal field theory techniques could be applied to other terms of the brane effective action. In order to keep the discussion as general as possible, we adopt here a ten-dimensional notation. Later, in sections 3 and 4 we will rephrase our findings using a four-dimensional language suitable to discuss compactifications of Type IIB string theory to $d = 4$.

At the lowest order, the fermionic interaction terms can be derived from disk 3-point correlators involving two vertices describing massless open-string fermions and one closed string vertex describing the background flux, as represented in figure 1. At a brane intersection massless open string modes can arise either from open strings starting and ending on the same stack of D-branes, or from open strings connecting two different sets of branes. In the former case the open string fields satisfy the standard untwisted boundary conditions and the corresponding vertex operators transform in the adjoint representation of the gauge group. In the latter case the string coordinates satisfy twisted boundary conditions characterized by twist parameters ϑ and the associated vertices carry Chan-Paton factors in the bi-fundamental representation of the gauge group; by inserting twisted open string vertices, one splits the disk boundary into different portions distinguished by their boundary conditions and Chan-Paton labels, see figure 1*b)*. We now give some details on these boundary conditions and later describe the physical vertex operators and their interactions with R-R and NS-NS background fluxes.

2.1 Boundary conditions and reflection matrices

The boundary conditions for the bosonic coordinates x^M $(M = 0, \ldots, 9)$ of the open string are given by

$$\Big(\delta_{MN}\,\partial_\sigma x^N + \mathrm{i}(\mathcal{F}_\sigma)_{MN}\,\partial_\tau x^N\Big)\Big|_{\sigma=0,\pi} = 0\,, \tag{2.1}$$

where δ_{MN} is the flat background metric[2] and

$$(\mathcal{F}_\sigma)_{MN} = B_{MN} + 2\pi\alpha' \, (F_\sigma)_{MN} \tag{2.2}$$

with B_{MN} the anti-symmetric tensor of the NS-NS sector and $(F_\sigma)_{MN}$ the background gauge field strength at the string end points $\sigma = 0, \pi$. Introducing the complex variable $z = \mathrm{e}^{\tau + \mathrm{i}\sigma}$ and the reflection matrices

$$R_\sigma = \big(1 - \mathcal{F}_\sigma\big)^{-1} \big(1 + \mathcal{F}_\sigma\big)\,, \tag{2.3}$$

the conditions (2.1) become

$$\overline{\partial} x^M \Big|_{\sigma=0,\pi} = (R_\sigma)^M_{\ N}\, \partial x^N \Big|_{\sigma=0,\pi}\,. \tag{2.4}$$

The standard Neumann boundary conditions (i.e. $R_\sigma = 1$) are obtained by setting $\mathcal{F}_\sigma = 0$, whereas the Dirichlet case (i.e. $R_\sigma = -1$) is recovered in the limit $\mathcal{F}_\sigma \to \infty$. A convenient way to solve (2.4) is to define multi-valued holomorphic fields $X^M(z)$ such that

$$X^M(\mathrm{e}^{2\pi \mathrm{i}} z) = \big(R_\pi^{-1}\, R_0\big)^M_{\ \ N}\, X^N(z) \equiv R^M_{\ N}\, X^N(z) \tag{2.5}$$

where $R \equiv R_\pi^{-1} R_0$ is the monodromy matrix. Then, putting the branch cut just below the negative real axis of the z-plane, the conditions (2.4) are solved by

$$x^M(z, \overline{z}) = q^M + \frac{1}{2}\Big[X^M(z) + \big(R_0\big)^M_{\ \ N}\, X^N(\overline{z})\Big]\,, \tag{2.6}$$

where z is restricted to the upper half-complex plane, and q^M are constant zero-modes.

For simplicity, in this paper we take the reflection matrices R_0 and R_π to be commuting. Then, with a suitable SO(10) transformation we can simultaneously diagonalize both matrices and write

$$R_\sigma = \mathrm{diag}\left(\mathrm{e}^{2\pi \mathrm{i}\theta^1_\sigma}, \mathrm{e}^{-2\pi \mathrm{i}\theta^1_\sigma} \ldots, \mathrm{e}^{2\pi \mathrm{i}\theta^5_\sigma}, \mathrm{e}^{-2\pi \mathrm{i}\theta^5_\sigma}\right)\,, \tag{2.7a}$$

$$R = \mathrm{diag}\left(\mathrm{e}^{2\pi \mathrm{i}\vartheta^1}, \mathrm{e}^{-2\pi \mathrm{i}\vartheta^1}, \ldots, \mathrm{e}^{2\pi \mathrm{i}\vartheta^5}, \mathrm{e}^{-2\pi \mathrm{i}\vartheta^5}\right)\,, \tag{2.7b}$$

with $\vartheta^I = \theta^I_0 - \theta^I_\pi$. In this basis the resulting (complex) coordinates, denoted by Z^I and $\overline{Z}^I$ with[3] $I = 1, \ldots, 5$, satisfy

$$\partial Z^I(\mathrm{e}^{2\pi \mathrm{i}} z) = \mathrm{e}^{2\pi \mathrm{i}\vartheta^I}\, \partial Z^I(z) \quad \text{and} \quad \partial \overline{Z}^I(\mathrm{e}^{2\pi \mathrm{i}} z) = \mathrm{e}^{-2\pi \mathrm{i}\vartheta^I}\, \partial \overline{Z}^I(z)\,, \tag{2.8}$$

and hence have an expansion in powers of $z^{n+\vartheta^I}$ and $z^{n-\vartheta^I}$, respectively, with $n \in \mathbb{Z}$. The corresponding oscillators act on a twisted vacuum $|\vec{\vartheta}\rangle$ created by the twist operator $\sigma_{\vec{\vartheta}}(z)$, which is a conformal field of dimension $h_{\sigma_{\vec{\vartheta}}} = \frac{1}{2}\sum_I \vartheta^I(1-\vartheta^I)$ satisfying the following OPE

$$\sigma_{\vec{\vartheta}}(z)\, \sigma_{-\vec{\vartheta}}(w) \sim (z-w)^{\sum_I \vartheta^I (1-\vartheta^I)}\,. \tag{2.9}$$

[2]Here, for convenience, we assume the space-time to have an Euclidean signature. Later, in section 3 we revert to a Minkowskian signature when appropriate.

[3]In the subsequent sections we will take the space-time to be the product of a four-dimensional part and an internal six-dimensional part. For notational convenience, we label the complex coordinates of the four-dimensional part by $I = 4, 5$ and those of the internal six-dimensional part by $I \equiv i = 1, 2, 3$.

For our purposes it is necessary to consider also the boundary conditions on the fermionic fields ψ^M of the open superstring in the RNS formalism, which are

$$\psi^M(\mathrm{e}^{2\pi\mathrm{i}}z) = \eta\, R^M_{\ N}\, \psi^N(z) \tag{2.10}$$

where $\eta = 1$ in the NS sector and $\eta = -1$ in the R sector. In the complex basis these boundary conditions become

$$\Psi^I(\mathrm{e}^{2\pi\mathrm{i}}z) = \eta\, \mathrm{e}^{2\pi\mathrm{i}\vartheta^I}\Psi^I(z) \quad \text{and} \quad \overline{\Psi}^I(\mathrm{e}^{2\pi\mathrm{i}}z) = \eta\, \mathrm{e}^{-2\pi\mathrm{i}\vartheta^I}\overline{\Psi}^I(z)\ . \tag{2.11}$$

Thus, in the NS sector Ψ^I and $\overline{\Psi}^I$ admit an expansion in powers of $z^{n+\vartheta^I}$ and $z^{n-\vartheta^I}$, respectively, with $n \in \mathbb{Z}$, so that their oscillators are of the type $\psi^I_{n+\vartheta^I+\frac{1}{2}}$. In the R sector they have a mode expansion in powers of $z^{n+\vartheta^I}$ and $z^{n-\vartheta^I}$, respectively, with $n \in \mathbb{Z}+\frac{1}{2}$. Note that if $\vartheta^I \neq 0$ neither the NS nor the R sector possesses zero-modes and the corresponding fermionic vacuum is non-degenerate. On the other hand if $\vartheta^I = 0$ there are zero-modes in the R sector, while if $\vartheta^I = \frac{1}{2}$ there are zero-modes in the NS sector. In these cases the corresponding fermionic vacuum is degenerate and carries the spinor representation of the rotation group acting on the directions in which the ϑ's vanish. For this reason it is in general necessary to determine the boundary reflection matrices also in the spinor representation, which we will denote by $\mathcal{R}_\sigma$.

To find these matrices, we first note that in the vector representation R_σ simply describes the product of five rotations with angles $2\pi\theta^I_\sigma$ in the five complex planes defined by the complex coordinates Z^I and $\overline{Z}^I$, as is clear from (2.7a). Then, recalling that the infinitesimal generator of such rotations in the spinor representation is $\frac{\mathrm{i}}{2}\Gamma^{I\bar{I}} = \frac{\mathrm{i}}{4}\left[\Gamma^I, \Gamma^{\bar{I}}\right]$ with Γ^I being the SO(10) Γ-matrices in the complex basis (see appendix A.1 for our conventions), we easily conclude that

$$\mathcal{R}_\sigma = \pm \prod_{I=1}^{5} \mathrm{e}^{\mathrm{i}\pi\theta^I_\sigma\Gamma^{I\bar{I}}} = \pm \prod_{I=1}^{5} \frac{\left(1 + \mathrm{i} f^I_\sigma \Gamma^{I\bar{I}}\right)}{\sqrt{1+(f^I_\sigma)^2}} \tag{2.12}$$

where $f^I_\sigma = \tan \pi\theta^I_\sigma$ and the overall sign depends on whether we have a D-brane or an anti D-brane. This general formula is particularly useful to derive the explicit expression for $\mathcal{R}_\sigma$ in the limits $f^I_\sigma \to 0$ or $f^I_\sigma \to \infty$ corresponding, respectively, to Neumann or Dirichlet boundary conditions in the I-plane. For example, for an open string starting from a Dp-brane extending in the directions $(01\ldots p)$ we have

$$\mathcal{R}_0 = \prod_{I=1}^{\frac{9-p}{2}} \left(\mathrm{i}\Gamma^{I\bar{I}}\right) = \Gamma^{(p+1)} \cdots \Gamma^9\ . \tag{2.13}$$

Being particular instances of rotations, the reflection matrices in the vector and spinor representations satisfy the following relation

$$\mathcal{R}_\sigma^{-1}\Gamma^M\mathcal{R}_\sigma = (R_\sigma)^M_{\ N}\Gamma^N\ . \tag{2.14}$$

2.2 Open and closed string vertices

A generic brane intersection can describe different physical situations depending on the values of the five twists ϑ^I.

When $\vartheta^I = 0$ for all I's, all fields are untwisted: this is the case of the open strings starting and ending on the same stack of D-branes which account for dynamical gauge excitations in the adjoint representation when the branes are space-filling, or for neutral instanton moduli when the branes are instantonic.

When $\vartheta^4 = \vartheta^5 = 0$ but the ϑ^i's with $i = 1, 2, 3$ are non vanishing, only the string coordinates in the space-time directions are untwisted and describe open strings stretching between different stacks of D-branes. The corresponding excitations organize in multiplets that transform in the bi-fundamental representation of the gauge group and always contain massless chiral fermions. When suitable relations among the non-vanishing twists are satisfied (e.g. $\vartheta^1 + \vartheta^2 + \vartheta^3 = 2$) also massless scalars appear in the spectrum and they can be combined with the fermions to form $\mathcal{N} = 1$ chiral multiplets suitable to describe the matter content of brane-world models.

Finally, when $\vartheta^4 = \vartheta^5 = \frac{1}{2}$, the string coordinates have mixed Neumann-Dirichlet boundary conditions in the last four directions and correspond to open strings connecting a space-filling D-brane with an instantonic brane. In this situation, if the ϑ^i's ($i = 1, 2, 3$) are vanishing, the instantonic brane describes an ordinary gauge instanton configuration and the twisted open strings account for the charged instanton moduli of the ADHM construction [17 − 20]; if instead also the ϑ^i's are non vanishing the instantonic branes represent exotic instantons of truly stringy nature whose role in the effective low-energy field theory has been recently the subject of intense investigation [26 − 47]. From these considerations it is clear that by considering open strings that are generically twisted we can simultaneously treat all configurations that are relevant for the applications mentioned in the introduction.

Open string vertices. Let us now focus on the R sector of the open strings at a generic brane intersection. Here the vertex operator for the lowest fermionic excitation $\Theta_{\mathcal{A}}$ is

$$V_\Theta(z) = \mathcal{N}_\Theta\, \Theta_{\mathcal{A}} \big[\sigma_{\vec{\vartheta}}\, s_{\vec{\epsilon}_{\mathcal{A}}+\vec{\vartheta}}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}\, k\cdot X}\big](z) \tag{2.15}$$

where we understand that the momentum k is defined only in untwisted directions. In this expression the index $\mathcal{A} = 1, \ldots, 16$ labels a spinor representation of SO(10) with definite chirality and runs over all possible choices of signs in the weight vector

$$\vec{\epsilon}_{\mathcal{A}} = \frac{1}{2}\Big(\pm, \pm, \pm, \pm, \pm\Big) \tag{2.16}$$

with, say, an odd number of $+$'s, and the symbol $s_{\vec{q}}(z)$ stands for the fermionic spin field

$$s_{\vec{q}}(z) = \mathrm{e}^{\mathrm{i}\sum_I q^I \varphi^I(z)} \tag{2.17}$$

where $\varphi^I(z)$ are the fields that bosonize the world-sheet fermions according to $\Psi^I = \mathrm{e}^{\mathrm{i}\varphi^I}$ (up to cocycle factors). Finally, $\phi(z)$ is the boson entering the superghost fermionization formulas, $\sigma_{\vec{\vartheta}}(z)$ is the bosonic twist field introduced above and $\mathcal{N}_\Theta$ is a normalization factor which will be discussed in the following sections.

The conformal weight of the vertex operator (2.15) is

$$h = \frac{k^2}{2} + \frac{1}{2}\sum_I \left[|\vartheta^I|(1-|\vartheta^I|) + (\epsilon^I_{\mathcal{A}} + \vartheta^I)^2\right] + \frac{3}{8} = \frac{k^2}{2} + 1 + \frac{1}{2}\sum_I \left(|\vartheta^I| + 2\vartheta^I \epsilon^I_{\mathcal{A}}\right) \quad (2.18)$$

and hence V_Θ describes a physical massless fermion $h = 1$, $k^2 = 0$, when the last term vanishes. This condition restricts the number of the allowed polarization components of Θ as follows

$$\Theta_{\mathcal{A}} \neq 0 \quad \text{only if} \quad \epsilon^I_{\mathcal{A}} = \begin{cases} \pm\frac{1}{2} & \text{for} \ \ \vartheta^I = 0 \\ -\frac{1}{2} & \text{for} \ \ \vartheta^I > 0 \\ \ \ \frac{1}{2} & \text{for} \ \ \vartheta^I < 0 \end{cases} \quad (2.19)$$

For example, when all ϑ^I's are vanishing we have a chiral spinor in ten dimensions but if only $\vartheta^4 = \vartheta^5 = 0$ we have a chiral spinor in the four untwisted directions along the (Z^4, Z^5) complex plane. On the other hand, in the instantonic brane constructions mentioned above, for which $\vartheta^4 = \vartheta^5 = \frac{1}{2}$, we see from the second line in (2.19) that the R sector describes fermions that do not carry a spinor index under Lorentz rotations along the ND four-dimensional plane, in perfect agreement with the ADHM realization of the charged fermionic instanton moduli.

Closed string vertices. We now describe the closed string vertex operators corresponding to background fluxes. In the closed string sector all fields (both bosonic and fermionic) are untwisted due to the periodic boundary conditions.[4] However, in the presence of D-branes a suitable identification between the left and the right moving components of the closed string has to be enforced at the boundary and a non-trivial dependence on the angles θ^I_σ appears through the matrices R_σ or $\mathcal{R}_\sigma$.

Let us first consider the R-R sector of the Type IIB theory, where the physical vertex operators for the field strengths of the anti-symmetric tensor fields are, in the $(-\frac{1}{2}, -\frac{1}{2})$ superghost picture,

$$V_F(z, \overline{z}) = \mathcal{N}_F \, F_{\mathcal{AB}} \, \mathrm{e}^{-\mathrm{i}\pi\alpha' k_{\mathrm{L}}\cdot k_{\mathrm{R}}} \left[s_{\vec{\epsilon}_{\mathcal{A}}} \, \mathrm{e}^{-\frac{1}{2}\phi} \, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{L}}\cdot X}\right](z) \times \left[\widetilde{s}_{\vec{\epsilon}_{\mathcal{B}}} \, \mathrm{e}^{-\frac{1}{2}\widetilde{\phi}} \, \mathrm{e}^{\mathrm{i}\, k_{\mathrm{R}}\cdot \widetilde{X}}\right](\overline{z}) \ . \quad (2.20)$$

In this expression $\mathcal{N}_F$ is a normalization factor that will be discussed later, k_{L} and k_{R} are the left and right momenta, and the tilde sign denotes the right-moving components. Furthermore, the factor $\mathrm{e}^{-\mathrm{i}\pi\alpha' k_{\mathrm{L}}\cdot k_{\mathrm{R}}}$ is a cocycle that allows for an off-shell extension of the closed string vertex with $k_{\mathrm{L}} \neq k_{\mathrm{R}}$, as discussed in ref. [58]. The bi-spinor polarization $F_{\mathcal{AB}}$ comprises all R-R field strengths of the Type IIB theory according to

$$F_{\mathcal{AB}} = \sum_{n=1,3,5} \frac{1}{n!} F_{M_1 \ldots M_n} \left(\Gamma^{M_1 \ldots M_n}\right)_{\mathcal{AB}} \ , \quad (2.21)$$

[4] Even if in later sections we will consider an orbifold compactification, we will include background fluxes from the untwisted closed string sector only. The study of the effect of background fluxes from twisted sectors of the orbifold theory is left to future work.

even if in our applications only the 3-form part will play a role. In the presence of D-branes the left and right moving components of the vertex operator V_F must be identified using the reflection rules discussed above. In practice (see for example ref. [58] for more details) this amounts to set

$$\widetilde{X}^M(\overline{z}) = (R_0)^M_{\ N}\, X^N(\overline{z})\,, \quad \widetilde{s}_{\vec{\epsilon}_\mathcal{A}}(\overline{z}) = (\mathcal{R}_0)^\mathcal{A}_{\ \mathcal{B}}\, s_{\vec{\epsilon}_\mathcal{B}}(\overline{z})\,, \quad \widetilde{\phi}(\overline{z}) = \phi(\overline{z}) \tag{2.22}$$

and modify the cocycle factor in the vertex operator (2.20) to $\mathrm{e}^{-\mathrm{i}\pi\alpha' k_\mathrm{L}\cdot(k_\mathrm{R} R_0)}$. As a consequence of the identifications (2.22), the R-R field-strength $F_{\mathcal{A}\mathcal{B}}$ gets replaced by the bi-spinor polarization $(F\,\mathcal{R}_0)_{\mathcal{A}\mathcal{B}}$ that incorporates also the information on the type of boundary conditions enforced by the D-branes.

Let us now turn to the NS-NS sector of the closed string. Here it is possible to write an effective BRST invariant vertex operator for the derivatives of the anti-symmetric tensor B that are related to the 3-form flux H. In the $(0,-1)$ superghost picture,[5] this effective vertex is

$$V_H(z,\overline{z}) = \mathcal{N}_H\left(\partial_M B_{NP}\right)\mathrm{e}^{-\mathrm{i}\pi\alpha' k_\mathrm{L}\cdot k_\mathrm{R}}\left[\psi^M\psi^N\mathrm{e}^{\mathrm{i}\,k_\mathrm{L}\cdot X}\right](z)\times\left[\widetilde{\psi}^P\,\mathrm{e}^{-\widetilde{\phi}}\,\mathrm{e}^{\mathrm{i}\,k_\mathrm{R}\cdot\widetilde{X}}\right](\overline{z}) \tag{2.23}$$

where again we have introduced a normalization factor and a cocycle. When we insert this vertex in a disk diagram, we must identify the left and right moving sectors using the reflection rules (2.22) supplemented by

$$\widetilde{\psi}^M(\overline{z}) = (R_0)^M_{\ N}\psi^N(\overline{z})\ . \tag{2.24}$$

Consequently, in (2.23) the polarization (∂B) is effectively replaced by $(\partial B R_0)$. Notice that the NS-NS polarization combines with the boundary reflection matrix in the vector representation R_0, in contrast to the R-R case where one finds instead the reflection matrix in the spinor representation $\mathcal{R}_0$.

2.3 The string correlator with R-R and NS-NS fluxes

We now evaluate the string correlation functions among two massless open string fermions and the background closed string flux, as represented in figure 1. It is a mixed open/closed string amplitude on a disk which, generically, has mixed boundary conditions. From the conformal field theory point of view such fermionic correlation functions are similar to the mixed amplitudes considered in refs. [55 – 58]. Let us analyze first the interaction with the R-R flux.

R-R flux. We take two fermionic open string vertices (2.15) and one closed string R-R vertex (2.20), and compute the amplitude

$$\mathcal{A}_F = \Big\langle V_\Theta(x)\,V_F(z,\overline{z})\,V_{\Theta'}(y)\Big\rangle = c_F\ \Theta_{\mathcal{A}_1}(F\mathcal{R}_0)_{\mathcal{A}_2\mathcal{A}_3}\,\Theta'_{\mathcal{A}_4}\ \times\ A^{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4} \tag{2.25}$$

where the prefactor

$$c_F = \mathcal{C}_{(p+1)}\,\mathcal{N}_\Theta\,\mathcal{N}_{\Theta'}\,\mathcal{N}_F\,, \tag{2.26}$$

[5] This particular asymmetric picture is chosen in view of the calculations of the disk amplitudes described in section 2.3.

accounts for the normalizations of the vertex operators and the topological normalization $\mathcal{C}_{(p+1)}$ of any disk amplitude with the boundary conditions of a Dp-brane [20, 59], whose explicit expression will be given in section 3.4 for D3-branes and D-instantons, see eqs. (3.45) and (3.50). The last factor in (2.25) is the 4-point correlator

$$A^{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4} = \int \frac{\prod_{i=1}^4 dz_i}{dV_{\rm CKG}}\, \mathrm{e}^{-\mathrm{i}\pi\alpha' k_2\cdot k_3} \Big\langle \prod_{i=1}^{4} \big[\sigma_{\vec\vartheta_i}\, s_{\vec\epsilon_i+\vec\vartheta_i}\, \mathrm{e}^{-\frac{1}{2}\phi}\, \mathrm{e}^{\mathrm{i}\, k_i\cdot X}\big](z_i)\Big\rangle \tag{2.27}$$

where we have used the convenient notation

$$\begin{aligned} &z_1 = x\,, \quad z_2 = z\,, \quad z_3 = \overline{z}\,, \qquad z_4 = y\,,\\ &k_1 = k\,, \quad k_2 = k_{\rm L}\,, \quad k_3 = k_{\rm R} R_0\,, \quad k_4 = k'\,,\\ &\vec\vartheta_1 = \vec\vartheta\,, \quad \vec\vartheta_2 = 0\,, \quad \vec\vartheta_3 = 0\,, \qquad \vec\vartheta_4 = -\vec\vartheta\,, \end{aligned} \tag{2.28}$$

and we have set $\vec\epsilon_i \equiv \vec\epsilon_{\mathcal{A}_i}$. Since the closed string vertex is untwisted, the two open string vertices must have opposite twists in order to have a non-vanishing amplitude. This explains the third line above, which, according to eq. (2.19), implies that when $\vartheta^I \neq 0$ the polarizations $\Theta_{\mathcal{A}_1}$ and $\Theta'_{\mathcal{A}_4}$ are not vanishing only if $\epsilon_1^I = -\epsilon_4^I$. Therefore, if $\vartheta^I \neq 0$ for all I's, the spinor weights $\vec\epsilon_1$ and $\vec\epsilon_4$ have different GSO parity and the amplitude (2.25) ceases to exist. To avoid this, from now on we will assume that at least one of the ϑ^I's be vanishing.[6] The evaluation of the correlator in (2.27) can be simplified by assuming that the closed string vertex does not carry momentum in the twisted directions (i.e. $k_2^I = k_3^I = 0$ if $\vartheta^I \neq 0$). This is not a restrictive choice for our purposes, since we will be interested in the effects induced by *constant* background fluxes.

In the correlator (2.27) the open string positions z_1 and z_4 are integrated on the real axis while the closed string variables z_2 and z_3 are integrated in the upper half complex plane, modulo the $\mathrm{Sl}(2;\mathbb{R})$ projective invariance that is fixed by the conformal Killing group volume $dV_{\rm CKG}$. Using this fact we have

$$\frac{\prod_{i=1}^4 dz_i}{dV_{\rm CKG}} = d\omega\,(1-\omega)^{-2}\,\big(z_{14}z_{23}\big)^2 \tag{2.29}$$

where ω is the anharmonic ratio

$$\omega = \frac{z_{12}z_{34}}{z_{13}z_{24}} \qquad (\,|\omega| = 1\,) \tag{2.30}$$

with $z_{ij} = z_i - z_j$. Due to our kinematical configuration, the contribution of the twist fields and the bosonic exponentials to the correlator (2.27) can be factorized and becomes

$$\Big\langle \sigma_{\vec\vartheta}(z_1)\,\sigma_{-\vec\vartheta}(z_4)\Big\rangle \Big\langle \prod_{i=1}^{4} \mathrm{e}^{\mathrm{i}\, k_i\cdot X(z_i)}\Big\rangle = z_{14}^{\sum_I \vartheta^I(1-\vartheta^I)}\;\omega^{\alpha' t}\,(1-\omega)^{\alpha' s} \tag{2.31}$$

[6] As pointed out in ref. [24] when all five ϑ^I's are non vanishing, the simplest tree-level diagram involving massless fermions of the twisted R sector requires at least three different types of boundary conditions and thus it is not of the type of amplitudes we are discussing here, which involve only two boundary changing operators.

where we have used (2.9), introduced the two kinematic invariants

$$s=(k_1+k_4)^2=(k_2+k_3)^2 \quad \text{and} \quad t=(k_1+k_3)^2=(k_2+k_4)^2\,, \tag{2.32}$$

and understood the momentum conservation.

Also the contribution of the spin fields and the superghosts can be easily evaluated using the bosonization formulas, that allow to write

$$\Big\langle \prod_{i=1}^{4} s_{\vec{\epsilon}_i+\vec{\vartheta}_i}(z_i)\,\mathrm{e}^{-\frac{1}{2}\phi(z_i)}\Big\rangle = \Big\langle \prod_{i=1}^{4} s_{\vec{\epsilon}_i}(z_i)\,\mathrm{e}^{-\frac{1}{2}\phi(z_i)}\Big\rangle \times \prod_{i<j} z_{ij}^{\vec{\epsilon}_i\cdot\vec{\vartheta}_j+\vec{\epsilon}_j\cdot\vec{\vartheta}_i+\vec{\vartheta}_i\cdot\vec{\vartheta}_j}\;. \tag{2.33}$$

The first factor in the right hand side is the four fermion correlator of the Type IIB superstring in ten dimensions which has been computed for example in ref. [60], namely

$$\Big\langle \prod_{i=1}^{4} s_{\vec{\epsilon}_i}(z_i)\,\mathrm{e}^{-\frac{1}{2}\phi(z_i)}\Big\rangle = \frac{1}{2}\prod_{i<j} z_{ij}^{-1}\Big[z_{13}z_{24}\big(\Gamma_M\big)^{\mathcal{A}_1\mathcal{A}_4}\big(\Gamma^M\big)^{\mathcal{A}_2\mathcal{A}_3}+z_{14}z_{23}\big(\Gamma_M\big)^{\mathcal{A}_1\mathcal{A}_3}\big(\Gamma^M\big)^{\mathcal{A}_2\mathcal{A}_4}\Big] \tag{2.34}$$

where we have understood the "charge" conservation $\sum_i \vec{\epsilon}_i = 0$. Furthermore, the ϑ-dependent factor in (2.33) can be simplified using the relations

$$\vec{\epsilon}_2\cdot\vec{\vartheta} = -\vec{\epsilon}_3\cdot\vec{\vartheta}\,, \quad \vec{\epsilon}_1\cdot\vec{\vartheta} = -\vec{\epsilon}_4\cdot\vec{\vartheta} = \frac{1}{2}\sum_I \vartheta^I\,, \tag{2.35}$$

that follow from (2.19) and the "charge" conservation of the spinor weights. Indeed, using (2.35) we have

$$\prod_{i<j} z_{ij}^{\vec{\epsilon}_i\cdot\vec{\vartheta}_j+\vec{\epsilon}_j\cdot\vec{\vartheta}_i+\vec{\vartheta}_i\cdot\vec{\vartheta}_j} = z_{14}^{\sum_I \vartheta^I(\vartheta^I-1)}\;\omega^{-\vec{\epsilon}_3\cdot\vec{\vartheta}}\;. \tag{2.36}$$

Collecting everything we find that the amplitude (2.27) can be written as

$$A^{\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3\mathcal{A}_4} = \big(\Gamma_M\big)^{\mathcal{A}_1\mathcal{A}_4}\big(\Gamma^M I_1\big)^{\mathcal{A}_2\mathcal{A}_3} + \big(\Gamma_M I_2\big)^{\mathcal{A}_1\mathcal{A}_3}\big(\Gamma^M\big)^{\mathcal{A}_2\mathcal{A}_4} \tag{2.37}$$

where we have introduced the two $\vec{\vartheta}$-dependent diagonal matrices with entries

$$\begin{aligned}\big(I_1\big)_{\mathcal{A}_3}^{\;\mathcal{A}_3} &= \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\int_\gamma d\omega\,(1-\omega)^{\alpha' s-1}\,\omega^{\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3-1}\,,\\ \big(I_2\big)_{\mathcal{A}_3}^{\;\mathcal{A}_3} &= \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\int_\gamma d\omega\,(1-\omega)^{\alpha' s}\,\omega^{\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3-1}\,,\end{aligned} \tag{2.38}$$

where $\mathcal{A}_3$ is the spinor index corresponding to the spinor weight $\vec{\epsilon}_3$. Here the integrals run around the clockwise oriented unit circle $\gamma : |\omega| = 1$, and can be evaluated to be [58]

$$\begin{aligned}\big(I_1\big)_{\mathcal{A}_3}^{\;\mathcal{A}_3} &= \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\Big(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3\right)}-1\Big)\,B\big(\alpha' s;\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3\big)\,,\\ \big(I_2\big)_{\mathcal{A}_3}^{\;\mathcal{A}_3} &= \frac{1}{2}\,\mathrm{e}^{-\frac{\mathrm{i}\pi\alpha' s}{2}}\Big(\mathrm{e}^{-2\pi\mathrm{i}\left(\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3\right)}-1\Big)\,B\big(\alpha' s+1;\alpha' t-\vec{\vartheta}\cdot\vec{\epsilon}_3\big)\,,\end{aligned} \tag{2.39}$$

where $B(a;b)$ is the Euler β-function. Plugging (2.37) into (2.25), with some simple manipulations we find

$$\mathcal{A}_F = -c_F\Big[\Theta'\Gamma^M\Theta\;\mathrm{tr}\big(F\mathcal{R}_0 I_1\Gamma_M\big) + \Theta'\Gamma^M F\mathcal{R}_0 I_2\Gamma_M\Theta\Big] \tag{2.40}$$

where the trace is understood in the 16×16 block spanned by the spinor indices $\mathcal{A}_i$'s. This expression can be further simplified by expanding the matrices $F\mathcal{R}_0 I_1$ and $F\mathcal{R}_0 I_2$ as

$$\left(F\mathcal{R}_0 I_a\right)_{\mathcal{A}\mathcal{B}} = \sum_{n=1,3,5} \frac{1}{n!} \left(F\mathcal{R}_0 I_a\right)_{N_1\ldots N_n} \left(\Gamma^{N_1\ldots N_n}\right)_{\mathcal{A}\mathcal{B}} \qquad (a=1,2)\,, \tag{2.41}$$

and by using the Γ-matrix identities

$$\begin{aligned} \mathrm{tr}\left(\Gamma^M\Gamma^N\right) &= 16\,\delta^{MN}\,, \quad \mathrm{tr}\left(\Gamma^M\Gamma^{N_1N_2N_3}\right) = \mathrm{tr}\left(\Gamma^M\Gamma^{N_1\ldots N_5}\right) = 0\,, \\ \Gamma_M\Gamma^{N_1\ldots N_n}\Gamma^M &= (-1)^n(10-2n)\,\Gamma^{N_1\ldots N_n}\ . \end{aligned} \tag{2.42}$$

After some straightforward algebra we find

$$\mathcal{A}_F = -8c_F\Theta'\Gamma^M\Theta\left[F\mathcal{R}_0(2I_1-I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\left[F\mathcal{R}_0 I_2\right]_{MNP}\,. \tag{2.43}$$

This formula is one of the main results of this section. It describes the tree-level bilinear fermionic couplings induced by R-R fluxes on a general brane intersection.

NS-NS flux. Let us now turn to the fermionic couplings induced by the NS-NS 3-form flux effectively described by the vertex operator (2.23). Such couplings arise from the following mixed disk amplitude

$$\mathcal{A}_H = \Big\langle V_\Theta(x)\,V_H(z,\overline{z})\,V_{\Theta'}(y)\Big\rangle = c_H\ \Theta_\mathcal{A}(\partial BR_0)_{MNP}\,\Theta'_\mathcal{B}\ \times\ A^{\mathcal{A}\mathcal{B};MNP} \tag{2.44}$$

where the normalization factor is

$$c_H = \mathcal{C}_{(p+1)}\,\mathcal{N}_\Theta\,\mathcal{N}_{\Theta'}\,\mathcal{N}_H \tag{2.45}$$

and the 4-point correlator is

$$\begin{aligned} A^{\mathcal{A}\mathcal{B};MNP} &= \int \frac{\prod_{i=1}^4 dz_i}{dV_{\mathrm{CKG}}}\ \mathrm{e}^{-\mathrm{i}\pi\alpha' k_2\cdot k_3}\Big\langle\sigma_{\vec{\vartheta}}(z_1)\,\sigma_{-\vec{\vartheta}}(z_4)\Big\rangle\Big\langle\prod_{i=1}^4 \mathrm{e}^{\mathrm{i}\,k_i\cdot X(z_i)}\Big\rangle \\ &\times\Big\langle s_{\vec{\epsilon}_\mathcal{A}+\vec{\vartheta}}(z_1)\,\psi^M\psi^N(z_2)\,\psi^P(z_3)\,s_{\vec{\epsilon}_\mathcal{B}-\vec{\vartheta}}(z_4)\Big\rangle\Big\langle\mathrm{e}^{-\frac{1}{2}\phi(z_1)}\mathrm{e}^{-\phi(z_3)}\mathrm{e}^{-\frac{1}{2}\phi(z_4)}\Big\rangle\,. \end{aligned} \tag{2.46}$$

Here we have used a notation similar to that of eq. (2.28) for the bosonic and twist fields, whose contribution is the same as in eq. (2.31) because of our kinematical configuration. Due to the Lorentz structure of the fermionic correlator, the second line of (2.46) can be written as

$$\begin{aligned} &\Big\langle s_{\vec{\epsilon}_\mathcal{A}+\vec{\vartheta}}(z_1)\,\psi^M\psi^N(z_2)\,\psi^P(z_3)\,s_{\vec{\epsilon}_\mathcal{B}-\vec{\vartheta}}(z_4)\Big\rangle\Big\langle\mathrm{e}^{-\frac{1}{2}\phi(z_1)}\mathrm{e}^{-\phi(z_3)}\mathrm{e}^{-\frac{1}{2}\phi(z_4)}\Big\rangle \\ &\qquad = f(z_{ij})\big(\Gamma^{MNP}\big)^{\mathcal{A}\mathcal{B}} + g(z_{ij})\Big[\delta^{MP}\big(\Gamma^N\big)^{\mathcal{A}\mathcal{B}} - \delta^{NP}\big(\Gamma^M\big)^{\mathcal{A}\mathcal{B}}\Big] \end{aligned} \tag{2.47}$$

where the two functions f and g can be determined, for example, by using the bosonization technique. If we pick a configuration such that the field $\psi^M\psi^N(z_2)$ can be bosonized as $\mathrm{e}^{\mathrm{i}\vec{\epsilon}_2\cdot\vec{\varphi}}$ with weight vectors of the form

$$\vec{\epsilon}_2 = \big(0,\ldots,\pm1,\ldots,\pm1,\ldots,0\big)\,, \tag{2.48}$$

corresponding to roots of SO(10), we can use the same strategy we have described before in the R-R case to find

$$f(z_{ij}) = \prod_{i<j} z_{ij}^{-1} \times \left(z_{14}z_{23}\right) \times z_{14}^{\sum_I \vartheta^I(\vartheta^I-1)}\ \omega^{-\vec{\epsilon}_3\cdot\vec{\vartheta}} ,$$
$$g(z_{ij}) = \prod_{i<j} z_{ij}^{-1} \times \left(z_{12}z_{34} + z_{13}z_{24}\right) \times z_{14}^{\sum_I \vartheta^I(\vartheta^I-1)}\ \omega^{-\vec{\epsilon}_3\cdot\vec{\vartheta}} , \tag{2.49}$$

where the last factors are the same as in eq. (2.35) and $\vec{\epsilon}_3$ is the weight vector in the vector representation associated to $\psi^P(z_3)$, of the form

$$\vec{\epsilon}_3 = \left(0, \ldots, \pm 1, \ldots, 0\right) . \tag{2.50}$$

Collecting everything, and introducing the diagonal matrices (with vector indices) $(I_1)^P{}_P$ and $(I_2)^P{}_P$ defined analogously to eq. (2.38), after some simple manipulations we obtain

$$\mathcal{A}_H = -4c_H\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial BR_0(2I_1 - I_2)\right]_{[MN]P} + 2c_H\Theta'\Gamma^{MNP}\Theta\left[\partial BR_0 I_2\right]_{MNP} \tag{2.51}$$

which is the NS-NS counterpart of the R-R amplitude (2.43) on a generic D brane intersection and shares with it the same type of fermionic structures.

3. Flux couplings with untwisted open strings ($\vec{\vartheta} = 0$)

We now exploit the results obtained in the previous section to analyze how constant background fluxes couple to untwisted open strings, i.e. strings starting and ending on a single stack of D-branes. This corresponds to set $\vec{\vartheta} = 0$ in all previous formulas which drastically simplify. Note that the condition $\vec{\vartheta} = 0$ implies that $\vec{\theta}_0 = \vec{\theta}_\pi$, so that the reflection rules are the same at the two string end-points. We can distinguish two cases, namely when these reflection rules are just signs (i.e. $\theta^I_\sigma = 0$ or 1) and when they instead depend on generic angles θ^I_σ. In the first case the branes are unmagnetized, while the second corresponds to magnetized branes.

Since we are interested in constant background fluxes, we can set the momentum of the closed string vertices to zero; this corresponds to take the limit $s = -2t \to 0$ in the integrals (2.39) which yields

$$2I_1 = I_2 = -\mathrm{i}\pi . \tag{3.1}$$

Using this result in the R-R and NS-NS amplitudes (2.43) and (2.51), we see that the fermionic couplings with a single Γ matrix vanish and only the terms with three Γ's survive, so that the total flux amplitude is

$$\mathcal{A} \equiv \mathcal{A}_F + \mathcal{A}_H = -2\pi\mathrm{i}\,\Theta\Gamma^{MNP}\Theta\left[\frac{c_F}{3}\left(F\mathcal{R}_0\right)_{MNP} + c_H\left(\partial BR_0\right)_{MNP}\right] . \tag{3.2}$$

Here we used the fact that the untwisted fermions Θ and Θ' in (2.43) and (2.51) actually describe the same field and only differ because they carry opposite momentum. For this reason we multiplied the above amplitudes by a symmetry factor of 1/2 and dropped the $'$ without introducing ambiguities.

It is clear from eq. (3.2) that once the flux configuration is given, the structure of the fermionic couplings for different types of D-branes depends crucially on the boundary reflection matrices R_0 and $\mathcal{R}_0$. Notice that the R-R piece of the amplitude (3.2) is generically non zero for 1-form, 3-form and 5-form fluxes. However, from now on we will restrict our analysis only to the 3-form and hence the bi-spinor to be used is simply

$$F_{\mathcal{AB}} = \frac{1}{3!} F_{MNP} \left(\Gamma^{MNP}\right)_{\mathcal{AB}} \ . \tag{3.3}$$

We can now specify better the normalization factors c_F and c_H. In fact the vertex (2.20) for a R-R 3-form and the NS-NS vertex (2.23) should account for the following quadratic terms of the bulk theory in the ten-dimensional Einstein frame:

$$\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g_{(E)}} \left(\frac{1}{3!} \, \mathrm{e}^{\varphi} F^2 + \frac{1}{3!} \, \mathrm{e}^{-\varphi} dB^2 \right) , \tag{3.4}$$

where φ is the dilaton and κ_{10} is the gravitational Newton constant in ten dimensions. In order to reproduce the above dilaton dependence, the normalization factors $\mathcal{N}_F$ and $\mathcal{N}_H$ of the R-R and NS-NS vertex operators must scale with the string coupling $g_s = \mathrm{e}^{\varphi}$ as

$$\mathcal{N}_F \sim g_s^{1/2} \quad \text{and} \quad \mathcal{N}_H \sim g_s^{-1/2} \, , \tag{3.5}$$

so that from eqs. (2.26) and (2.45) we obtain

$$c_H = c_F / g_s \ . \tag{3.6}$$

Taking all this into account, we can rewrite the total amplitude (3.2) as

$$\mathcal{A} \equiv \mathcal{A}_F + \mathcal{A}_H = -\frac{2\pi \mathrm{i}}{3!} \, c_F \, \Theta \Gamma^{MNP} \Theta \, T_{MNP} \tag{3.7}$$

where

$$T_{MNP} = \left(F\mathcal{R}_0\right)_{MNP} + \frac{3}{g_s} \left(\partial B R_0\right)_{[MNP]} \ . \tag{3.8}$$

Up to now we have used a ten-dimensional notation. However, since we are interested in studying the flux induced couplings for gauge theories and instantons in four dimensions, it becomes necessary to split the indices $M, N, \ldots = 0, 1, \ldots, 9$ appearing in the above equations into four-dimensional space-time indices $\mu, \nu, \ldots = 0, 1, 2, 3$, and six-dimensional indices $m, n, \ldots = 4, 5, \ldots, 9$ labeling the directions of the internal space (which we will later take to be compact, for example a 6-torus $\mathcal{T}_6$ or an orbifold thereof). Clearly background fluxes carrying indices along the space-time break the four-dimensional Lorentz invariance and generically give rise to deformed gauge theories. Effects of this kind have already been studied using world-sheet techniques in refs. [55, 57] where a non vanishing R-R 5-form background of the type $F_{\mu\nu mnp}$ was shown to originate the $\mathcal{N} = 1/2$ gauge theory, and in ref. [21] where the so-called Ω deformation of the $\mathcal{N} = 2$ gauge theory was shown to derive from a R-R 3-form flux of the type $F_{\mu\nu m}$. In the following, however, we will consider only internal fluxes, like F_{mnp} or $(\partial B)_{mnp}$, which preserve the four-dimensional Lorentz invariance, and the fermionic amplitudes we will compute are of the form

$$\mathcal{A} = -\frac{2\pi \mathrm{i}}{3!} \, c_F \, \Theta \Gamma^{mnp} \Theta \, T_{mnp} \tag{3.9}$$

We now analyze the structure of these couplings beginning with the simplest case of space-filling unmagnetized D-branes; later we examine unmagnetized Euclidean branes and finally branes with a non-trivial world-volume magnetic field.

3.1 Unmagnetized D-branes

Even if the fermionic couplings (3.7) have been derived in section 2 assuming a Euclidean signature, when we discuss space-filling D-branes with $\vec{\vartheta} = 0$, the rotation to a Minkowskian signature poses no problems. In this case, Θ becomes a Majorana-Weyl spinor in ten dimensions which in particular satisfies

$$\Theta\Gamma^{mnp}\Theta = -\left(\Theta\Gamma^{mnp}\Theta\right)^* \ . \tag{3.10}$$

Furthermore for an unmagnetized Dp-brane that fills the four-dimensional Minkowski space and possibly extends also in some internal directions, the reflection matrices R_0 and $\mathcal{R}_0$ are very simple: indeed in the vector representation

$$R_0 = \mathrm{diag}(\pm 1, \pm 1, \ldots) \,, \tag{3.11}$$

where the entries specify whether a direction is longitudinal $(+)$ or transverse $(-)$, while in the spinor representation

$$\mathcal{R}_0 = \Gamma^{p+1} \cdots \Gamma^9 \ . \tag{3.12}$$

Using these matrices we easily see that T_{mnp} is a *real* tensor, so that in view of eq. (3.10) also the total fermionic amplitude (3.9) is real, as it should be.

The explicit expression of T_{mnp} is particularly simple in the case of brane configurations which respect the $4+6$ structure of the space-time, i.e. D3- and D9- branes. For space-filling D3-branes all internal indices are transverse, so that $R_0\big|_{\rm int} = -1$ and $\mathcal{R}_0 = \Gamma^4 \cdots \Gamma^9$. From eq. (3.8) it follows then

$$T_{mnp} = (*_6 F)_{mnp} - \frac{1}{g_s}\, H_{mnp} \tag{3.13}$$

where $*_6$ denotes the Poincarè dual in the six-dimensional internal space and $H = dB$.[7]

For D9-branes, instead, all internal indices are longitudinal, and to emphasize this fact we denote them as $\hat{m}, \hat{n}, \ldots$ In this case we simply have $R_0 = 1$ and $\mathcal{R}_0 = 1$ so that

$$T_{\hat{m}\hat{n}\hat{p}} = F_{\hat{m}\hat{n}\hat{p}} + \frac{1}{g_s}\, H_{\hat{m}\hat{n}\hat{p}} \ . \tag{3.14}$$

Note however that D9-branes must always be accompanied by orientifold 9-planes (O9) for tadpole cancellation and that the corresponding orientifold projection kills the NS-NS flux $H_{\hat{m}\hat{n}\hat{p}}$. If we take this fact into account, the coupling tensor for D9-branes reduces to

$$T_{\hat{m}\hat{n}\hat{p}} = F_{\hat{m}\hat{n}\hat{p}} \ . \tag{3.15}$$

[7] In our conventions $(*_6 F)_{mnp} = \frac{1}{3!}\epsilon_{mnprst}\, F^{rst}$ and $H_{mnp} = 3\partial_{[m}B_{np]} = \left(\partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}\right)$.

	0	1	2	3	4	5	6	7	8	9	T_{mnp}
D3	–	–	–	–	×	×	×	×	×	×	$(*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp}$
D5	–	–	–	–	–	–	×	×	×	×	$\frac{1}{g_s}\, H_{\hat{m}\hat{n}p}$; $-\frac{1}{2}\, F_{\hat{m}}{}^{qr} \epsilon_{qrnp}$; $-\frac{1}{g_s}\, H_{mnp}$
D7	–	–	–	–	–	–	–	–	×	×	$F_{\hat{m}\hat{n}}{}^{q}\, \epsilon_{qp} + \frac{1}{g_s}\, H_{\hat{m}\hat{n}p}$
D9	–	–	–	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

Table 1: Structure of the fermionic couplings T induced by background fluxes on D3, D5, D7 and D9-branes after taking into account the appropriate orientifold projections; longitudinal internal directions are labeled by $\hat{m}, \hat{n}, \ldots$ and internal transverse ones by $m, n, \ldots$

The case of space-filling D7- and D5-branes is slightly more involved since for these branes the internal directions are partially longitudinal and partially transverse. In particular, for D7-branes the longitudinal internal indices $\hat{m}, \hat{n} \ldots$ take four values while the transverse indices $p, q, \ldots$ take two values. eq. (3.8) implies then that the only non vanishing components of the T tensor for D7-branes are

$$T_{\hat{m}\hat{n}\hat{p}} = \frac{1}{g_s}\, H_{\hat{m}\hat{n}\hat{p}}\,, \quad T_{\hat{m}\hat{n}p} = F_{\hat{m}\hat{n}}{}^{q}\, \epsilon_{qp} + \frac{1}{g_s}\, H_{\hat{m}\hat{n}p} \quad \text{and} \quad T_{\hat{m}np} = -\frac{1}{g_s}\, H_{\hat{m}np}\,. \tag{3.16}$$

If one introduces O7-planes to cancel the tadpoles produced by the D7-branes, one can see that the corresponding orientifold projection[8] $\Omega I_2 (-1)^{F_L}$ removes all F and H components with an even number of transverse indices so that the only surviving couplings are

$$T_{\hat{m}\hat{n}p} = F_{\hat{m}\hat{n}}{}^{q}\, \epsilon_{qp} + \frac{1}{g_s}\, H_{\hat{m}\hat{n}p}\ . \tag{3.17}$$

For D5-branes the situation is somehow complementary, since the longitudinal internal indices take two values while the transverse ones run over four values. In this case one can show that the non vanishing components of the T tensor are

$$T_{\hat{m}\hat{n}p} = \frac{1}{g_s}\, H_{\hat{m}\hat{n}p}\,, \quad T_{\hat{m}np} = -\frac{1}{2}\, F_{\hat{m}}{}^{qr} \epsilon_{qrnp} - \frac{1}{g_s}\, H_{\hat{m}np} \quad \text{and} \quad T_{mnp} = -\frac{1}{g_s}\, H_{mnp}\ . \tag{3.18}$$

Again the O5-planes required for tadpole cancellation enforce an orientifold projection ΩI_4 which removes the components of $H(F)$ with an even(odd) number of transverse indices. Thus, the coupling $T_{\hat{m}np}$ reduces to

$$T_{\hat{m}np} = -\frac{1}{2}\, F_{\hat{m}}{}^{qr} \epsilon_{qrnp}\ . \tag{3.19}$$

The fermionic couplings for the various D-branes we have discussed, taking into account the appropriate orientifold projections, are summarized in table 1.

[8] The extra $(-1)^{F_L}$ appearing in the case of O3/O7-planes ensure that the corresponding orientifold actions square to one, i.e. $(\Omega I_{4n+2}(-1)^{F_L})^2 = I_{4n+2}^2 (-1)^{F_L+F_R} = 1$.

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These results clearly exhibit the fact that the R-R and NS-NS 3-form fluxes do not appear on equal footing in the effective couplings T. This is due to the different $\mathcal{R}_0$ and R_0 reflection matrices entering in the definition of the R-R and NS-NS vertex operators as discussed in section 2. It is interesting to observe in table 1 that, while for D9- and D5-branes the fermionic couplings depend either on F or on H, for D3- and D7-branes they depend on a combination of the R-R and NS-NS fluxes. This follows from the fact that O3- and O7-planes act on the same way on R-R and NS-NS 3-forms. By introducing the complex 3-form[9]

$$G = F - \frac{\mathrm{i}}{g_s} H \,, \tag{3.20}$$

it is possible to rewrite the D3 brane coupling (3.13) as

$$T_{mnp} = (*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp} = \mathrm{Re}\left(*_6 G - \mathrm{i} G\right)_{mnp} \,. \tag{3.21}$$

Thus our explicit conformal field theory calculation confirms that an imaginary self-dual (ISD) 3-form flux G does not couple to unmagnetized D3-branes, a well-known result that has been previously obtained using purely supergravity methods [10, 12, 48, 50, 52].

Also the fermionic couplings (3.17) for the D7 branes can be written in terms of the 3-form flux G. Indeed, introducing a complex notation and denoting as i and $\bar{\imath}$ the complex directions of the plane transverse to the D7-branes (sometimes in the literature also called D7$_i$-branes), we have

$$T_{\hat{m}\hat{n}i} = \mathrm{i}\, G_{\hat{m}\hat{n}i} \quad \text{and} \quad T_{\hat{m}\hat{n}\bar{\imath}} = -\mathrm{i}\, G^*_{\hat{m}\hat{n}\bar{\imath}} \,, \tag{3.22}$$

in agreement with the structure of soft fermionic mass terms found in ref. [12].

3.2 Unmagnetized Euclidean branes

Euclidean branes that are transverse to the four-dimensional space-time and extend partially or totally in the internal directions are relevant to discuss non-perturbative instanton effects in the framework of branes models. In this case, to treat consistently the flux induced couplings it is necessary to work in a space with Euclidean signature as we have done in section 2. Then, the massless fermions Θ cannot satisfy a Majorana condition, and relations like (3.10) do not hold any more. On the other hand, in Euclidean space there is no issue about the reality of a fermionic amplitude and, as we will see, also the coupling tensor T is in general complex.

Let us begin by considering the D-instantons (or D(-1)-branes) for which all ten directions are transverse. In this case we have

$$R_0 = -1 \quad \text{and} \quad \mathcal{R}_0 = \Gamma^0\Gamma^1 \cdots \Gamma^9 \equiv \mathrm{i}\,\Gamma^{\mathrm{E}}_{(11)} \tag{3.23}$$

[9]Self-duality of type IIB can be used to promote this expression to its SL$(2,\mathbb{Z})$-covariant version $G = F - \tau H$ with $\tau = C_0 - \mathrm{i}e^{-\varphi}$. A direct evaluation of the C_0-dependent term however requires a string amplitude involving two closed and two open string insertions in the disk.

where $\Gamma^{\rm E}_{(11)}$ is the chirality matrix in ten Euclidean dimensions. Thus, recalling our chirality choice for the spinors Θ, we easily see that for D-instantons the T tensor (3.8) is simply

$$T_{mnp} = -{\rm i}\, F_{mnp} - \frac{1}{g_s}\, H_{mnp} = -{\rm i}\, G_{mnp} \ . \tag{3.24}$$

Let us now turn to Euclidean instantonic 5-branes (or E5-branes) extending in the six internal directions. In this case the reflection matrix in the vector representation entering in the fermionic coupling T is $R_0|_{\rm int} = 1$ along the internal directions while the matrix in the spinor representation is

$$\mathcal{R}_0 = \Gamma^0\Gamma^1\Gamma^2\Gamma^3 = -{\rm i}\,\Gamma^4\cdots\Gamma^9\Gamma^{\rm E}_{(11)} \ . \tag{3.25}$$

Therefore, for unmagnetized E5-branes we obtain from eq. (3.8)

$$T_{\hat m\hat n\hat p} = {\rm i}\,(*_6F)_{\hat m\hat n\hat p} + \frac{1}{g_s}\, H_{\hat m\hat n\hat p} \tag{3.26}$$

where we have used the same index notation introduced in the previous subsection. The above coupling simply reduces to

$$T_{\hat m\hat n\hat p} = {\rm i}\,(*_6F)_{\hat m\hat n\hat p} \tag{3.27}$$

in an orientifold model with O9-planes.

In the literature some attention has been devoted also to Euclidean 3-branes (or E3-branes) extending along four of the six internal directions [51, 53]. These branes have some similarity with the D7-branes considered in the previous subsection, and thus our discussion can follow the same path. Using again the convention of splitting the internal indices into longitudinal (hatted) and transverse (unhatted) ones, we can show that the flux-induced fermionic couplings on E3-branes are

$$T_{\hat m\hat n\hat p} = \frac{1}{g_s}\, H_{\hat m\hat n\hat p}\,, \quad T_{\hat m\hat n p} = -\frac{\rm i}{2}\,\epsilon_{\hat m\hat n\hat r\hat s}\, F^{\hat r\hat s}{}_p + \frac{1}{g_s}\, H_{\hat m\hat n p} \quad \text{and} \quad T_{\hat m n p} = -\frac{1}{g_s}\, H_{\hat m n p} \ . \tag{3.28}$$

If we consider the appropriate orientifold projections, which in this case remove both $H_{\hat m\hat n\hat p}$ and $H_{\hat m np}$, we see that the only non-vanishing coupling is

$$T_{\hat m\hat n p} = -\frac{\rm i}{2}\,\epsilon_{\hat m\hat n\hat r\hat s}\, F^{\hat r\hat s}{}_p + \frac{1}{g_s}\, H_{\hat m\hat n p} \ . \tag{3.29}$$

This is in perfect agreement with the result of refs. [51, 53] that has been derived with pure supergravity methods. To make the comparison easier, we observe that the E3-fermionic terms can be rewritten as

$$\Theta\Gamma^{\hat m\hat n p}\Theta\, T_{\hat m\hat n p} = \Theta\Gamma^{\hat m\hat n p}\, \widetilde G_{\hat m\hat n p}\,\Theta \tag{3.30}$$

where

$$\widetilde G = \frac{1}{g_s}\, H + {\rm i}\, F\,\gamma_{(5)} \tag{3.31}$$

	0	1	2	3	4	5	6	7	8	9	T_{mnp}
D(−1)	×	×	×	×	×	×	×	×	×	×	$-\mathrm{i}F_{mnp} - \frac{1}{g_s}H_{mnp}$
E1	×	×	×	×	−	−	×	×	×	×	$\frac{1}{g_s}\,H_{\hat{m}\hat{n}p}$; $-\,\mathrm{i}\,\epsilon_{\hat{m}\hat{q}}\,F^{\hat{q}}{}_{np}$; $-\,\frac{1}{g_s}\,H_{mnp}$
E3	×	×	×	×	−	−	−	−	×	×	$-\frac{\mathrm{i}}{2}\,\epsilon_{\hat{m}\hat{n}\hat{r}\hat{s}}\,F^{\hat{r}\hat{s}}{}_{p} + \frac{1}{g_s}\,H_{\hat{m}\hat{n}p}$
E5	×	×	×	×	−	−	−	−	−	−	$\mathrm{i}\,(*_6 F)_{\hat{m}\hat{n}\hat{p}}$

Table 2: Structure of the fermionic couplings T induced by background fluxes on D(−1), E1, E3 and E5 instantonic branes after taking into account the appropriate orientifold projections; the longitudinal internal directions are labeled by $\hat{m}, \hat{n}, \ldots$, the internal transverse ones by $m, n, \ldots$.

is the flux combination that is usually introduced in this case, with $\gamma_{(5)}$ being the chirality matrix for the four-dimensional brane world-volume. We further remark that our general formula (3.7) accounts for all flux-induced fermionic terms of the E3-brane effective action discussed in refs. [51, 53] including those which break the Lorentz invariance in the first four directions.

For completeness we also mention that the fermionic couplings for the Euclidean 1-branes (or E1-branes) are given by

$$T_{\hat{m}\hat{n}p} = \frac{1}{g_s}\,H_{\hat{m}\hat{n}p}\,, \quad T_{\hat{m}np} = -\mathrm{i}\,\epsilon_{\hat{m}\hat{q}}\,F^{\hat{q}}{}_{np} - \frac{1}{g_s}\,H_{\hat{m}np} \quad \text{and} \quad T_{mnp} = -\frac{1}{g_s}\,H_{mnp}\,; \tag{3.32}$$

note that $H_{\hat{m}np}$ is removed by the orientifold projection when the E1-branes are considered together with D5/D9-branes and the corresponding orientifold planes. The structure of the various fermionic couplings for the instantonic branes discussed above is summarized in table 2.

We conclude our analysis by observing that in presence of E-branes, the spacetime filling Dp-branes live in the Euclidean ten-dimensional space. Still, the couplings of such Dp-branes are again given by the same linear combinations of F and H like in the Minkowskian case considered in last section, since R_0 and $\mathcal{R}_0$ are trivial along the would be time direction.

3.3 Magnetized branes

The results of the previous subsections can be generalized in a rather straightforward way to branes with a non-trivial magnetization on their world-volume for which the longitudinal coordinates satisfy non-diagonal boundary conditions. Indeed we can start from the same brane configurations we have analyzed before, introduce a world-volume gauge field A that couples to the open string end-points and obtain a magnetization $\mathcal{F}_0 = \mathcal{F}_\pi = 2\pi\alpha'(dA)$. In this way we can use the same R-R and NS-NS background fluxes of the previous subsections and simply study the new couplings induced by the world-volume magnetization through the reflection matrices R_0 and $\mathcal{R}_0$ given in eqs. (2.3) and (2.12).

As an example we briefly discuss the case of the magnetized E5 branes which play an important role in the instanton calculus of the gauge theory engineered with wrapped

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D9-branes and O9-planes [23, 24]. Adopting the same index notation as before, one can easily realize that the spinor reflection matrix (2.12) for a magnetized E5 brane can be written in the real basis as

$$\mathcal{R}_0 = \Gamma^0 \cdots \Gamma^3 \,\mathcal{U}_0 = -\mathrm{i}\,\Gamma^4 \cdots \Gamma^9 \Gamma^{\mathrm{E}}_{(11)}\,\mathcal{U}_0 \tag{3.33}$$

where

$$\mathcal{U}_0 = \frac{1}{\sqrt{\det(1-\mathcal{F}_0)}}\ ;\, \mathrm{e}^{\frac{1}{2}(\mathcal{F}_0)_{\hat{m}\hat{n}}\Gamma^{\hat{m}\hat{n}}}\, ; \tag{3.34}$$

in which the symbol $;\cdots;$ means antisymmetrization on the vector indices of the Γ's, so that only a finite number of terms appear in the expansion of the exponential. In our case we explicitly have

$$\begin{aligned} ;\, \mathrm{e}^{\frac{1}{2}(\mathcal{F}_0)_{\hat{m}\hat{n}}\Gamma^{\hat{m}\hat{n}}}\, ;\ = 1 + \frac{1}{2}\,(\mathcal{F}_0)_{\hat{m}\hat{n}}\Gamma^{\hat{m}\hat{n}} + \frac{\mathrm{i}}{16}\,(\mathcal{F}_0)^{\hat{m}\hat{n}}(\mathcal{F}_0)^{\hat{p}\hat{q}}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}\hat{s}}\,\Gamma^{\hat{r}\hat{s}}\Gamma_{(7)} \\ - \frac{\mathrm{i}}{3!\cdot 8}(\mathcal{F}_0)^{\hat{m}\hat{n}}(\mathcal{F}_0)^{\hat{p}\hat{q}}(\mathcal{F}_0)^{\hat{r}\hat{s}}\epsilon_{\hat{m}\hat{n}\hat{p}\hat{q}\hat{r}\hat{s}}\,\Gamma_{(7)} \end{aligned} \tag{3.35}$$

where $\Gamma_{(7)} = \mathrm{i}\Gamma^4 \ldots \Gamma^9$ is the chirality matrix of the E5-brane world volume. Using this expression in eq. (3.8) and focusing for simplicity only on R-R fluxes since the NS-NS fluxes are anyhow removed by the orientifold projection, after simple manipulations we find that the fermionic couplings of a magnetized E5-brane are described by the tensor

$$\begin{aligned} T_{\hat{m}\hat{n}\hat{p}} = \frac{1}{\sqrt{\det(1-\mathcal{F}_0)}}\Big[&\mathrm{i}\,(*_6F)_{\hat{m}\hat{n}\hat{p}} + 3\mathrm{i}\,(*_6F)_{\hat{m}\hat{n}}{}^{\hat{q}}\,(\mathcal{F}_0)_{\hat{q}\hat{p}} \\ &+ \frac{3\mathrm{i}}{8}F_{\hat{m}\hat{n}}{}^{\hat{q}}(\mathcal{F}_0)^{\hat{r}\hat{s}}(\mathcal{F}_0)^{\hat{t}\hat{u}}\epsilon_{\hat{q}\hat{r}\hat{s}\hat{t}\hat{u}\hat{p}} - \frac{\mathrm{i}}{3!\,8}F_{\hat{m}\hat{n}\hat{p}}(\mathcal{F}_0)^{\hat{q}\hat{r}}(\mathcal{F}_0)^{\hat{s}\hat{t}}(\mathcal{F}_0)^{\hat{u}\hat{v}}\,\epsilon_{\hat{q}\hat{r}\hat{s}\hat{t}\hat{u}\hat{v}}\Big]\,. \end{aligned} \tag{3.36}$$

In the same way, and always starting from the general formula (3.8) one can discuss all other types of magnetized branes.

3.4 Flux-induced fermionic mass and lifting of instanton zero-modes

To complete the previous analysis we write the fermion bilinear $\Theta\Gamma^{mnp}\Theta$ using a four-dimensional spinor notation; in this way the structure of the flux-induced fermionic masses will be more clearly exposed. According to our 4+6 splitting, the anti-chiral ten dimensional spinor $\Theta_{\mathcal{A}}$ decomposes as

$$\Theta_{\mathcal{A}} \;\rightarrow\; \left(\Theta^{\alpha A}, \Theta_{\dot{\alpha}A}\right) \tag{3.37}$$

where α ($\dot{\alpha}$) are chiral (anti-chiral) indices in four dimensions, and the lower (upper) indices A are chiral (anti-chiral) spinor indices of the internal six dimensional space. Furthermore, by decomposing the Γ matrices according to

$$\Gamma^\mu = \gamma^\mu \otimes 1\,, \quad \Gamma^m = \gamma_{(5)} \otimes \gamma^m\,, \tag{3.38}$$

one can show that

$$\Theta\Gamma^{mnp}\Theta = -\mathrm{i}\,\Theta^{\alpha A}\Theta_{\alpha}{}^{B}\left(\overline{\Sigma}^{mnp}\right)_{AB} - \mathrm{i}\,\Theta_{\dot{\alpha}A}\Theta^{\dot{\alpha}}{}_{B}\left(\Sigma^{mnp}\right)^{AB} \tag{3.39}$$

$T^{\rm ISD}$	$\to$	$T_{(0,3)} \oplus T_{(1,2)_{\rm NP}} \oplus T_{(2,1)_{\rm P}} = \bar{\mathbf{1}} \oplus \bar{\mathbf{3}} \oplus \bar{\mathbf{6}}$
$T^{\rm IASD}$	$\to$	$T_{(3,0)} \oplus T_{(2,1)_{\rm NP}} \oplus T_{(1,2)_{\rm P}} = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{6}$

Table 3: Decomposition of the ISD and IASD parts of the 3-form T. The $(2,1)$ and $(1,2)$ components are distinguished into primitive (P) and non-primitive (NP) parts. The last column displays the SU(3) content of the various pieces.

where Σ^{mnp} and $\overline{\Sigma}^{mnp}$ are respectively the chiral and anti-chiral blocks of γ^{mnp} (see appendix A.1 for details). It is important to notice that

$$*_6\Sigma^{mnp} = -{\rm i}\,\Sigma^{mnp}\,, \quad *_6\overline{\Sigma}^{mnp} = +{\rm i}\,\overline{\Sigma}^{mnp} \tag{3.40}$$

so that Σ^{mnp} only couples to an imaginary self-dual (ISD) tensor, while $\overline{\Sigma}^{mnp}$ only couples to an imaginary anti-self dual (IASD) tensor. More explicitly, we have

$$\begin{aligned}\Theta\Gamma^{mnp}\Theta\, T_{mnp} &= -{\rm i}\,\Theta^{\alpha A}\Theta_\alpha{}^B\big(\overline{\Sigma}^{mnp}\big)_{AB}\, T^{\rm IASD}_{mnp} - {\rm i}\,\Theta_{\dot\alpha A}\Theta^{\dot\alpha}{}_B\big(\Sigma^{mnp}\big)^{AB}\, T^{\rm ISD}_{mnp}\\ &= -{\rm i}\,\Theta^{\alpha A}\Theta_\alpha{}^B T_{AB} - {\rm i}\,\Theta_{\dot\alpha A}\Theta^{\dot\alpha}{}_B T^{AB}\end{aligned} \tag{3.41}$$

where

$$T^{\rm ISD}_{mnp} = \frac{1}{2}\big(T - {\rm i}\,*_6 T\big)_{mnp}\,, \quad T^{\rm IASD}_{mnp} = \frac{1}{2}\big(T + {\rm i}\,*_6 T\big)_{mnp}\,. \tag{3.42}$$

In the second line of eq. (3.41) we have adopted a $\mathrm{SU}(4) \sim \mathrm{SO}(6)$ notation and defined the IASD and ISD parts of the T-tensor as the following 4×4 symmetric matrices

$$T_{AB} = \big(\overline{\Sigma}^{mnp}\big)_{AB}\, T^{\rm IASD}_{mnp}\,, \quad T^{AB} = \big(\Sigma^{mnp}\big)^{AB}\, T^{\rm ISD}_{mnp}\,, \tag{3.43}$$

with upper (lower) indices A, B running over the $\mathbf{4}$ ($\bar{\mathbf{4}}$) representations of SU(4). Fixing a complex structure, the 3-form tensors $T^{\rm ISD}, T^{\rm IASD}$ can be decomposed into their (3,0),(2,1),(1,2) and (0,3) parts as indicated in table 3. The $(2,1)$ components are distinguished into six primitive ones (P), satisfying $g^{j\bar k}T_{ij\bar k} = 0$, and three non-primitive ones (NP), satisfying $T_i = g^{j\bar k}T_{ij\bar k}$. A similar decomposition holds for the $(1,2)$ part. The various components transform in irreducible representations of the $\mathrm{SU}(3) \in \mathrm{SU}(4)$ holonomy group under which the internal coordinates $Z^i, \bar Z^i$ transform as $\mathbf{3}$ and $\bar{\mathbf{3}}$ respectively and spinors like $\mathbf{4} = \mathbf{1} + \mathbf{3}$ and $\bar{\mathbf{4}} = \bar{\mathbf{1}} + \bar{\mathbf{3}}$. The SU(3) content of the T-tensor is displayed in the last column in table 3.

Let us now use this information to rewrite the fermionic terms we have discussed in the previous subsections, focusing in particular on D3-branes and D-instantons on flat space. In the case of D3-branes, we can use a Minkowski signature and the Majorana-Weyl fermion Θ decomposes as in (3.39) where the four-dimensional chiral and anti-chiral components are related by charge conjugation and assembled into four Majorana spinors. These are the four gauginos leaving on the world-volume of the D3-brane, and for future notational convenience we will denote their chiral and anti-chiral parts as $\Lambda^{\alpha A}$ and $\bar\Lambda_{\dot\alpha A}$ (instead of $\Theta^{\alpha A}$ and $\Theta_{\dot\alpha A}$). Then, using eqs. (3.21) and (3.39) in the general expression (3.9), we obtain

$$\mathcal{A}_{\rm D3} = \frac{2\pi{\rm i}}{3!}\, c_F\, {\rm Tr}\Big[\Lambda^{\alpha A}\Lambda_\alpha{}^B\big(\overline{\Sigma}^{mnp}\big)_{AB}\, G^{\rm IASD}_{mnp} - \bar\Lambda_{\dot\alpha A}\bar\Lambda^{\dot\alpha}{}_B\big(\Sigma^{mnp}\big)^{AB}\,\big(G^{\rm IASD}_{mnp}\big)^*\Big] \tag{3.44}$$

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where we have made explicit the colour trace generators.[10]

Recalling that the topological normalization of any disk amplitude with D3-strings is [20]

$$\mathcal{C}_{(4)} = \frac{1}{\pi^2\, \alpha'^2\, g^2_{\rm YM}}\,, \tag{3.45}$$

one can show that in order to obtain gauginos with canonical dimension of (length)$^{-3/2}$ and standard kinetic term of the form

$$\frac{1}{g^2_{\rm YM}} \int d^4x\, {\rm Tr}\big(-\, 2{\rm i}\, \bar\Lambda_{\dot\alpha A} \bar{\not{\mathcal{D}}}^{\dot\alpha\beta} \Lambda^A_\beta\big)\,, \tag{3.46}$$

one has to normalize the gaugino vertices with

$$\mathcal{N}_\Lambda = (2\pi\alpha')^{\frac{3}{4}}\ . \tag{3.47}$$

Then, using these ingredients the prefactor appearing in eq. (3.44) becomes

$$c_F = \frac{4}{g^2_{\rm YM}}\, (2\pi\alpha')^{-\frac{1}{2}}\, \mathcal{N}_F\ . \tag{3.48}$$

From the explicit expression of the amplitude (3.44) we see that an IASD G-flux configuration induces a Majorana mass[11] for the gauginos leading to supersymmetry breaking on the gauge theory [48, 10−12]. Notice that the mass term for the two different chiralities are complex conjugate of each other: $T^{\rm IASD} = -{\rm i}\, G^{\rm IASD}$ and $T^{\rm ISD} = {\rm i}(G^{\rm IASD})^*$. This is a consequence of the Majorana condition that the four-dimensional spinors inherit from the Majorana-Weyl condition of the fermions in the original ten-dimensional theory.

If we decompose $G^{\rm IASD}$ as indicated in table 3, we see that a G-flux of type $(1,2)_{\rm P}$ gives mass to the three gauginos transforming non-trivially under SU(3) but keeps the SU(3)-singlet gaugino massless, thus preserving $\mathcal{N}=1$ supersymmetry. On the other hand, a G-flux of type $(3,0)$, or $(2,1)_{\rm NP}$ gives mass also to the SU(3)-singlet gaugino.

Things are rather different instead on D-instantons whose fermionic coupling is given by eq. (3.24). Indeed, by inserting such coupling in eq. (3.9) and using again eq. (3.39) we obtain

$$\mathcal{A}_{{\rm D}(-1)} = \frac{2\pi{\rm i}}{3!}\, c_F(\Theta) \Big[\, \Theta^{\alpha A} \Theta_\alpha{}^B \big(\overline{\Sigma}^{mnp}\big)_{AB}\, G^{\rm IASD}_{mnp} + \bar\Theta_{\dot\alpha A} \bar\Theta^{\dot\alpha}{}_B \big(\Sigma^{mnp}\big)^{AB}\, G^{\rm ISD}_{mnp} \Big] \tag{3.49}$$

where now the prefactor $c_F(\Theta)$ contains the topological normalization of the D(-1) disks (the value of the gauge instanton action), namely [20]

$$\mathcal{C}_{(0)} = \frac{8\pi^2}{g^2_{\rm YM}} \quad \Rightarrow \quad c_F(\Theta) = \frac{8\pi^2}{g^2_{\rm YM}} \mathcal{N}^2_\Theta\, \mathcal{N}_F \tag{3.50}$$

From the amplitude (3.49) we explicitly see that both the IASD and the ISD components of the G-flux couple to the D-instanton fermions; however the couplings are different and

[10] We use the following normalization: ${\rm Tr}\,\big(T^a T^b\big) = \frac{1}{2}\,\delta^{ab}$.

[11] Notice that this is the mass term for gauginos which are not canonically normalized, as we do not rescale away the overall factor of $1/g^2_{\rm YM}$ appearing in eq. (3.46).

independent for the two chiralities since they are not related by complex conjugation, as always in Euclidean spaces. In particular, comparing eqs. (3.44) and (3.49), we see that an ISD G-flux does not give a mass to any gauginos but instead induces a "mass" term for the anti-chiral instanton zero-modes which are therefore lifted. This effect may play a crucial role in discussing the non-perturbative contributions of the so-called "exotic" D-instantons for which the neutral anti-chiral zero modes $\bar{\Theta}_{\dot{\alpha}A}$ must be removed [30, 31] or lifted by some mechanism [36, 40]. Introducing an ISD G-flux is one of such mechanisms as we will discuss in more detail in section 6.

4. Flux couplings with twisted open strings ($\vec{\vartheta} \neq 0$)

As we have emphasized, the general world-sheet calculation presented in section 2 allows to obtain the couplings between closed string fluxes and open string fermions at a generic D-brane intersection, even for non-vanishing twist parameters $\vec{\vartheta}$. A systematic study of the amplitudes (2.43) and (2.51) when $\vec{\vartheta} \neq 0$ will be presented elsewhere; here we just analyze a simple case of such twisted amplitudes which will be relevant for the applications discussed in section 6.

The case we discuss is that of the 3-form flux couplings with the twisted fermions stretching between a D3-brane and a D-instanton which represent the charged (or flavored) fermionic moduli of the $\mathcal{N} = 4$ ADHM construction of instantons (see for example refs. [25, 20]) and are usually denoted as μ^A and $\bar{\mu}^A$ depending on the orientation. In the notation of section 2 the D3/D(-1) and D(-1)/D3 strings are characterized by twist vectors of the form

$$\vec{\vartheta} = \left(0, 0, 0, +\frac{1}{2}, +\frac{1}{2}\right) \quad \text{and} \quad \vec{\vartheta}' = \left(0, 0, 0, -\frac{1}{2}, -\frac{1}{2}\right) \tag{4.1}$$

respectively, and thus, according to eq. (2.19), the open string fermions in these sectors have weight vectors

$$\vec{\epsilon}_1 = \left(\vec{\epsilon}_A, -\frac{1}{2}, -\frac{1}{2}\right) \quad \text{and} \quad \vec{\epsilon}_4 = \left(\vec{\epsilon}_A, +\frac{1}{2}, +\frac{1}{2}\right). \tag{4.2}$$

The notation $\vec{\epsilon}_A$ ($\vec{\epsilon}^A$) denotes an anti-chiral (chiral) spinor weight of the internal SO(6) rotation group. The vertex operators corresponding to (4.2) (see eq. (2.15)) are then

$$V_\mu(z) = \mathcal{N}_\mu\, \mu^A \big[\sigma_{\vec{\vartheta}}\, s_{\vec{\epsilon}_A}\, \mathrm{e}^{-\frac{1}{2}\phi}\big](z) \quad \text{and} \quad V_{\bar{\mu}}(z) = \mathcal{N}_{\bar{\mu}}\, \bar{\mu}^A \big[\sigma_{\vec{\vartheta}'}\, s_{\vec{\epsilon}_A}\, \mathrm{e}^{-\frac{1}{2}\phi}\big](z) \tag{4.3}$$

where $\mathcal{N}_\mu, \mathcal{N}_{\bar{\mu}}$ are suitable normalizations. Notice that since the last two components of $(\vec{\epsilon}_1 + \vec{\vartheta})$ and $(\vec{\epsilon}_4 + \vec{\vartheta}')$ are zero, only a spin-field $s_{\vec{\epsilon}_A} = S_A$ of the internal SO(6) appears in the vertices (4.3); furthermore there is no momentum in any direction since $\mu^A, \bar{\mu}^A$ are moduli rather than dynamical fields. Note also that both μ^A and $\bar{\mu}^A$ carry the same SO(6) chirality.

On the other hand, the vertex operator for a R-R field strength contains two parts: one with left and right weights of the type

$$\vec{\epsilon}_2 = \left(\vec{\epsilon}^A, \vec{\epsilon}^{\dot{\alpha}}\right) \quad \text{and} \quad \vec{\epsilon}_3 = \left(\vec{\epsilon}^B, \vec{\epsilon}^{\dot{\beta}}\right), \tag{4.4}$$

and one with weights of the type

$$\vec{\epsilon}_2 = \big(\vec{\epsilon}_A, \vec{\epsilon}_\alpha\big) \quad \text{and} \quad \vec{\epsilon}_3 = \big(\vec{\epsilon}_B, \vec{\epsilon}_\beta\big)\,, \tag{4.5}$$

where $\vec{\epsilon}_\alpha$ ($\vec{\epsilon}^{\dot{\alpha}}$) are the chiral (anti-chiral) spinor weights[12] of SO(4). We now show that when the R-R field is an internal 3-form F_{mnp} only the part in (4.4) couples to the μ and $\bar{\mu}$'s.

Let us consider the general R-R amplitude (2.43) and take, for example, the $\sigma = 0$ boundary on the D(-1)-brane, i.e. $R_0 = -1$ and $\mathcal{R}_0 = \mathrm{i}\Gamma^{\mathrm{E}}_{(11)}$. Let us then observe that the spinor reflection matrix can be effectively replaced by $\mathcal{R}_0 = -\mathrm{i}$, since our GSO projection selects the anti-chiral sector, and that the two $\vec{\vartheta}\cdot\vec{\epsilon}_3$-dependent integrals I_1 and I_2, defined in (2.38), are scalars along the six internal directions because the internal components of $\vec{\vartheta}$ are vanishing (see eq. (4.1)). All this implies that the term with a single Γ in (2.43) vanishes, so that the only non-trivial contribution comes from the term with three Γ's.

To proceed we need to evaluate the integral I_2. In the limit $s = -2t \to 0$, from eq. (2.39) we easily find

$$\big(I_2\big)_{\mathcal{A}_3}^{\ \mathcal{A}_3} = \frac{1}{2\vec{\vartheta}\cdot\vec{\epsilon}_3}\left(1 - \mathrm{e}^{2\pi\mathrm{i}\vec{\vartheta}\cdot\vec{\epsilon}_3}\right)\,; \tag{4.6}$$

recall that $\mathcal{A}_3$ is the spinor index corresponding to the weight $\vec{\epsilon}_3$. There are two distinct cases, corresponding to the two possibilities (4.4) and (4.5) respectively. In the first case we have

$$\vec{\vartheta}\cdot\vec{\epsilon}_3 = 0 \qquad \Rightarrow \qquad \big(I_2\big)_{\mathcal{A}_3}^{\ \mathcal{A}_3} = -\pi\mathrm{i}\,, \tag{4.7}$$

while in the second case we have

$$\begin{aligned}
\vec{\vartheta}\cdot\vec{\epsilon}_3 &= +\frac{1}{2} \quad \text{if} \quad \vec{\epsilon}_\beta = \frac{1}{2}(++) \qquad \Rightarrow \qquad \big(I_2\big)_{\mathcal{A}_3}^{\ \mathcal{A}_3} = +2\,,\\
\vec{\vartheta}\cdot\vec{\epsilon}_3 &= -\frac{1}{2} \quad \text{if} \quad \vec{\epsilon}_\beta = \frac{1}{2}(--) \qquad \Rightarrow \qquad \big(I_2\big)_{\mathcal{A}_3}^{\ \mathcal{A}_3} = -2\,.
\end{aligned} \tag{4.8}$$

Using the explicit expression of the Γ matrices given in appendix A.1, it is not difficult to realize that the above results can be summarized by writing

$$I_2 = -\pi\mathrm{i}\left(\frac{1+\Gamma_{(7)}}{2}\right) - 2\mathrm{i}\,\Gamma^{01}\left(\frac{1-\Gamma_{(7)}}{2}\right) \tag{4.9}$$

where $\Gamma_{(7)}$ is the chirality matrix in the six-dimensional internal space. Indeed, restricting to the anti-chiral block, one can check that the first term in (4.9) accounts for the matrix elements (4.7), while the second term for the matrix elements (4.8). At this point it is clear that with an internal R-R 3-form flux F_{mnp}, the coefficient $\big(F\mathcal{R}_0 I_2\big)_{mnp}$ of the amplitude (2.43) can only receive contribution from the first term in (4.9), which yields

$$\mathcal{A}_F \sim \bar{\mu}^A \mu^B \left(\overline{\Sigma}^{mnp}\right)_{AB} F^{\mathrm{IASD}}_{mnp}\,. \tag{4.10}$$

The evaluation of coupling with an internal NS-NS 3-form flux H_{mnp} is much simpler. In fact the left and right weights appearing in the NS-NS vertex operator are

$$\vec{\epsilon}_2 = \big(\pm\vec{e}_m \pm \vec{e}_n, \vec{0}\big) \quad \text{and} \quad \vec{\epsilon}_3 = \big(\pm\vec{e}_p, \vec{0}\big)\,, \tag{4.11}$$

[12] In our conventions, as explained in appendix A.1, $\alpha \in \{\frac{1}{2}(++), \frac{1}{2}(--)\}$ and $\dot{\alpha} \in \{\frac{1}{2}(-+), \frac{1}{2}(+-)\}$.

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coordinates	h_0	h_1	h_2	h_3
Z^1	$+$	$+$	$-$	$-$
Z^2	$+$	$-$	$+$	$-$
Z^3	$+$	$-$	$-$	$+$

Table 4: Orbifold group action on the complex coordinates Z^i of $\mathcal{T}^6$.

with $\vec{e}_{m,n,p}$ unit vectors in the SO(6) weight space specifying the H_{mnp}-hyperplane. Thus, we always have $\vec{\theta} \cdot \vec{\epsilon}_3 = 0$ which implies that the entries of the two diagonal matrices I_1 and I_2 (with vector indices) are

$$2\left(I_1\right)^P_{\ P} = \left(I_2\right)^P_{\ P} = -\pi\mathrm{i}\ . \tag{4.12}$$

Thus, from eq. (2.51) we see that the term with a single Γ vanishes, while the term with three Γ's yields

$$\mathcal{A}_H \sim \bar{\mu}^A \mu^B \left(\overline{\Sigma}^{mnp}\right)_{AB} H^{\mathrm{IASD}}_{mnp}\ . \tag{4.13}$$

Collecting the two contributions (4.10) and (4.13) and reinstating the appropriate normalizations, we finally obtain

$$\mathcal{A}_{\mathrm{D3/D(-1)}} \equiv \mathcal{A}_F + \mathcal{A}_H = \frac{4\pi\mathrm{i}}{3!}\, c_F(\mu)\, \bar{\mu}^A \mu^B \left(\overline{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp} \tag{4.14}$$

where $c_F(\mu) = \mathcal{C}_{(0)}\, \mathcal{N}_\mu\, \mathcal{N}_{\bar{\mu}}\, \mathcal{N}_F$ with $\mathcal{C}_{(0)}$ given in eq. (3.50). Notice that no symmetry factors has to be included in this amplitude, since μ and $\bar{\mu}$ are really distinct and independent quantities. This amplitude together with the one in eq. (3.49) accounts for the flux induced fermionic couplings on the D-instanton effective action, and their meaning will be discussed in section 6.

5. Flux couplings in an $\mathcal{N} = 1$ orbifold set-up

The results of the previous sections clearly show that internal NS-NS and R-R fluxes bear important consequences on the brane effective action and may be relevant in phenomenological applications. Therefore it is particularly interesting to study such flux interactions in models with $\mathcal{N} = 1$ supersymmetry. To do so we adopt a toroidal orbifold compactification scheme where string theory remains calculable and the flux couplings are basically those described in section 3. We consider type IIB theory compactified on a Calabi-Yau 3-fold with the $\mathcal{N} = 2$ bulk supersymmetry further broken down to $\mathcal{N} = 1$ by the introduction of D-branes and O-planes. To be specific we consider the orbifold $\mathcal{T}_6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with $\mathcal{T}_6$ completely factorized as a product of three 2-torii. The action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on the orthonormal complex coordinates Z^i $(i = 1, 2, 3)$ of the torus is in table 4. In order to be self-contained we now briefly recall the structure of the closed string multiplets and the pattern of fractional D-branes that can be introduced in this orbifold.

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5.1 Closed and open string sectors in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Closed string states. Let us start by considering the oriented closed string states before the introduction of O-planes. The massless closed string states in the orbifold organize into a gravity multiplet, $h_{2,1}$ vector multiplets and $h_{1,1}+1$ hypermultiplets of the $\mathcal{N}=2$ supersymmetry. For strings defined on the quotient space, the orbifold projection onto $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant states has to be enforced and as usual we distinguish between untwisted and twisted sectors.

The untwisted sector follows from that on $\mathcal{T}^6$ after restricting to $\mathbb{Z}_2 \times \mathbb{Z}_2$-invariant components. It contains: the gravity multiplet; the universal hypermultiplet having as bosonic components the dilaton φ, the axion C_0 and the dualized NS-NS and R-R 2-forms B_2 and C_2 with four dimensional indices; $h_{21}^{\text{untw}} = 3$ vector multiplets with bosonic components (V^i, u^i) where the scalars u^i parametrize the complex structure deformations; $h_{11}^{\text{untw}} = 3$ hypermultiplets containing the scalars $(v_i, b_i, c_i, \tilde{c}^i)$ with v_i representing the Kähler parameter of the i-th torus, b_i and c_i the components of the NS-NS and R-R 2-forms along the i-th torus, and $\tilde{c}^i$ the R-R 4-form components along the dual 4-cycle.

Closed strings on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold have also twisted sectors associated to each of the three non trivial elements h_i and localized at the 16 possible fixed loci of their action; the total number of twisted sectors is therefore $3 \times 16 = 48$. To fully specify the orbifold model, we must declare the action of the group elements h_i also on the twist fields. There are two consistent possibilities, which correspond to the singular limits of two different CY manifolds with $(h_{11}, h_{21}) = (51, 3)$ and $(h_{11}, h_{21}) = (3, 51)$. Here we restrict ourselves to the first choice,[13] corresponding to take all twisted fields invariant under h_i. With this choice, the twisted sectors contribute to the massless spectrum with $h_{11}^{\text{tw}} = 48$ hypermultiplets containing the scalars $(v_{\hat{i}}, b_{\hat{i}}, c_{\hat{i}}, \tilde{c}^{\hat{i}})$, $\hat{i} = 1, \ldots 48$, which describe, respectively, the deformations of the blow-up modes of the vanishing 2-cycles and the exceptional components of the NS-NS 2-form and of the R-R 2- and 4-forms. It is important to recall that the orbifold limit is attained with a non-zero background value of the NS-NS B-field on the vanishing cycles [61], so that the scalar fields $b_{\hat{i}}$ mentioned above represent fluctuations around this value.

Finally the introduction of O-planes projects the spectrum onto the subset of ΩI-invariant states with Ω the worldsheet parity and I some involution of the CY threefold. The resulting spectrum falls into vectors and chiral multiplets of the unbroken $\mathcal{N}=1$ supersymmetry. The details depends on the choice of the O-planes. For example for a vacuum built out of O3/O7-planes, B_2 and C_2 are odd while for O5/O9 planes, B_2 and C_4 are odd. As a consequence, in the twisted sector either $b_{\hat{i}}$ and $c_{\hat{i}}$, or $b_{\hat{i}}$ and $\tilde{c}^{\hat{i}}$ would be projected out for the O3/O7 and O5/O9-choices respectively.

Open string states and fractional D-branes. The fundamental types of D-branes which can be placed transversely to an orbifold space are called fractional branes [62]. Such branes must be localized at one of the fixed points of the orbifold group (which in

[13]The second choice corresponds to declare that the twist field Δ_{ij} in the four-dimensional plane (ij) transforms in the representation $|\epsilon_{ijk}| R_k$.

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irrep R_A	fields		
R_0	A_μ	$\Lambda^0 \equiv \Lambda^{---}$	$\bar{\Lambda}_0 \equiv \bar{\Lambda}_{+++}$
R_1	ϕ^1	$\Lambda^1 \equiv \Lambda^{-++}$	$\bar{\Lambda}_1 \equiv \bar{\Lambda}_{+--}$
R_2	ϕ^2	$\Lambda^2 \equiv \Lambda^{+-+}$	$\bar{\Lambda}_2 \equiv \bar{\Lambda}_{-+-}$
R_3	ϕ^3	$\Lambda^3 \equiv \Lambda^{++-}$	$\bar{\Lambda}_3 \equiv \bar{\Lambda}_{--+}$

Table 5: Representation of the orbifold group on the $\mathcal{N} = 4$ open string fields

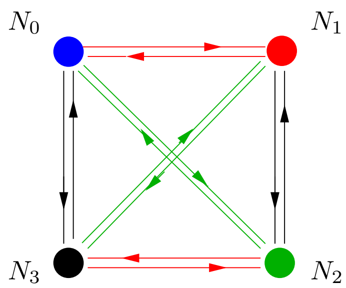

Figure 2: The quiver diagram encoding the field content and the charges for fractional D-branes of in the local orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. The dots represent the branes associated with the irrep R_A of the orbifold group. A stack of N_A such branes supports a $\mathrm{U}(N_A)$ gauge theory. An oriented link from the A-th to the B-th dot corresponds to a chiral multiplet ϕ^{AB} transforming in the $(N_A, \overline{N}_B)$ representation of the gauge group and in the $R_A\, R_B^{-1}$ representation of the orbifold group.

our case are 64). For simplicity we focus on fractional D3 branes sitting at a specific fixed point (say, the origin) and work around this configuration. Locally our system is undistinguishable from the theory living on the non-compact orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$.

The fractional branes are in correspondence with the irreducible representations R_A of the orbifold group: in fact the Chan-Paton indices of an open string connecting two fractional branes of type A and B transform in the representation $R_A \otimes R_B$. In addition the orbifold group acts on the SO(6) internal indices of the open string fields as indicated in table 5. This action should be compensated by that on the Chan-Paton indices in such a way that the whole field is $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant. For example the vector A_μ and the gaugino Λ^0 are invariant under the orbifold group and therefore they should carry indices $(N_A, \overline{N}_A)$ in the adjoint of the $\prod_{A=0}^{3} \mathrm{U}(N_A)$. The remaining fields fall into chiral multiplets transforming in the bifundamental representations $(N_A, \overline{N}_B)$. The resulting quiver diagram is displayed in figure 2.

To discuss the couplings of fractional branes to closed strings, it may be convenient to describe the branes by means of boundary states [63, 64], which we indicate schematically as $|A\rangle$ for a brane of type A. It turns out (see for example ref. [65]) that these boundary states $|A\rangle$ are suitable combinations of boundary states $|I\rangle\rangle$ associated to the h_I-twisted sector, namely

$$|A\rangle = \frac{1}{4} \sum_I \, (\mathrm{Ch})^I_A |I\rangle\rangle \ . \tag{5.1}$$

with $(\mathrm{Ch})^I_A = \mathrm{tr}_{R_A}(h_I)$. In our orbifold, using the character table (8) given in appendix A.2, these sums explicitly read [66]

$$\begin{aligned} |0\rangle &= \frac{1}{4}\Big(|0\rangle\!\rangle + |1\rangle\!\rangle + |2\rangle\!\rangle + |3\rangle\!\rangle\Big), \quad |1\rangle = \frac{1}{4}\Big(|0\rangle\!\rangle + |1\rangle\!\rangle - |2\rangle\!\rangle - |3\rangle\!\rangle\Big)\,, \\ |2\rangle &= \frac{1}{4}\Big(|0\rangle\!\rangle - |1\rangle\!\rangle + |2\rangle\!\rangle - |3\rangle\!\rangle\Big), \quad |3\rangle = \frac{1}{4}\Big(|0\rangle\!\rangle - |1\rangle\!\rangle - |2\rangle\!\rangle + |3\rangle\!\rangle\Big)\,. \end{aligned} \tag{5.2}$$

These boundary states show that the fractional D3-branes couple not only to twisted closed string fields but to *untwisted* ones as well, with a fractional tension and a fractional charge given by 1/4 of the ones of the regular branes.

The fractional branes corresponding to R_A $(A \neq 0)$ can also be interpreted geometrically as D5-branes suitably wrapped on exceptional[14] 2-cycles $e^{\hat{A}}$ in the blown-up space. To this extent, the background value of the NS-NS 2-form B_2 in the orbifold limit plays a crucial role in accounting for the untwisted couplings of the branes. We will take advantage of this description in the remaining of this section when we interpret the flux couplings computed in section 3 in the effective low-energy supergravity theory.

5.2 Gauge kinetic functions and soft supersymmetry breaking on D3-branes

In presence of D-branes the $\mathcal{N}=2$ bulk supersymmetry of the chosen compactification is reduced to a specific $\mathcal{N}=1$ slice depending on the boundary conditions imposed by the branes on the spin fields, which are encoded in the spinor reflection matrix $\mathcal{R}_0$ of eq. (2.12). The supersymmetry left unbroken by D-branes should be aligned to that preserved by O-planes and tadpole conditions should be enforced. As a consequence, the field content of the bulk theory is reorganized into $\mathcal{N}=1$ multiplets; in particular the compactification moduli, as well as the dilaton and axion fields, are assembled into complex scalars within suitable chiral superfields, which couple to the $\mathcal{N}=1$ vector and chiral multiplets living on the D-branes.

The tree-level effective action on the D-branes can be obtained in the field theory limit $\alpha' \to 0$ from disk diagrams and takes the standard form of an $\mathcal{N}=1$ supersymmetric action in which the couplings are actually functions of the moduli due to the possible interactions with closed string fields. In particular, the gauge Lagrangian depends on the bulk moduli M via its "gauge kinetic function" $f(M)$ which encodes the information on the Yang-Mills coupling g_{YM} and the θ-angle θ_{YM} according to

$$f(M) = \frac{\theta_{\mathrm{YM}}}{2\pi} + \mathrm{i}\,\frac{4\pi}{g^2_{\mathrm{YM}}}\,, \tag{5.3}$$

so that the quadratic part in the gauge field strengths reads

$$-\frac{1}{8\pi}\int d^4x\ \mathrm{Im}\,f(M)\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{8\pi}\int d^4x\ \mathrm{Re}\,f(M)\,\mathrm{Tr}\left(F_{\mu\nu}{}^*F^{\mu\nu}\right)\,. \tag{5.4}$$

Actually, the residual supersymmetry implies that the gauge Lagrangian takes the form

$$-\frac{\mathrm{i}}{8\pi}\int d^2\theta\ f\big(M(\theta)\big)\,\mathrm{Tr}\left(W^\alpha(\theta)W_\alpha(\theta)\right) + \mathrm{c.c.} \tag{5.5}$$

[14] Since we focus on branes localized at the origin, for each A we consider only one of the 16 possible exceptional cycles $e^{\hat{A}}$.

where $W^\alpha(\theta)$ is the $\mathcal{N}=1$ gauge superfield whose lowest component is the gaugino Λ^α, while the moduli M in the gauge kinetic function f get promoted to chiral superfields $M(\theta)$.

This is very interesting in two respects. First, the determination of the gauge kinetic functions for different types of branes preserving the same $\mathcal{N}=1$ supersymmetry suggests a way to assemble the bulk moduli and their superpartners into $\mathcal{N}=1$ chiral multiplets. Second, the Lagrangian (5.5) contains a gaugino mass term, which arises whenever the θ^2 component of $f\big(M(\theta)\big)$ assumes a non-zero vacuum expectation value. As we will see such mass terms can be related to the flux-induced fermionic couplings computed in section 3 (see in particular eq. (3.44)). To establish the precise relation we need to determine both the gauge kinetic functions for the D-branes used to engineer the gauge theory, and the appropriate complex combinations of the compactification moduli M that can be promoted to chiral superfields. This is what we do in the following.

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Gauge kinetic function. Let us take a fractional D3-brane, say of type A. To deduce its gauge kinetic function f_A we have several possibilities. We can derive the quadratic terms in the gauge fields of eq. (5.4) from disk diagrams, with the boundary attached to the brane and with two open string vertices for the gauge field inserted on the boundary and closed string scalar vertices in the interior. Alternatively, we can compute the coupling among closed strings and the boundary state $|A\rangle_F$ representing the fractional D3-brane with a constant magnetic field F turned on in the world-volume and infer from it the gauge kinetic function f_A (see e.g. ref. [67]). Finally, we can simply read off the coupling from the Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) actions of the fractional D3-brane.

The last option is viable if one regards the fractional D3-branes of type A as D5-branes wrapped[15] on the twisted 2-cycle $\hat{e}^A$, as recalled in section 5. In this case the DBI action with an additional Wess-Zumino term for the D5-brane (in the string frame) is

$$S_{D3,A} = -T_5 \int_{D3}\int_{\hat{e}^A} \mathrm{e}^{-\varphi}\sqrt{-\det\left(G+\mathcal{F}\right)} + T_5 \int_{D3}\int_{\hat{e}^A} \sum_{n=0}^{3} C_{2n}\, e^{\mathcal{F}} , \tag{5.6}$$

where $\mathcal{F} = B_2 + 2\pi\alpha' F$, and $T_5 = T_3/(4\pi^2\alpha')$ with $T_3 = (2\pi)^{-1}(2\pi\alpha')^{-2}$ being the D3-brane tension. Expanding to quadratic order in F and using the non-zero background value of B_2 along the vanishing cycles [61]

$$\int_{\hat{e}^A} B_2 = \frac{1}{4}(4\pi^2\alpha') \tag{5.7}$$

one finds

$$S_{D3,A} = -\frac{1}{16\pi}\int_{D3} \mathrm{e}^{-\varphi}\, F_{\mu\nu}F^{\mu\nu} + \frac{1}{16\pi}\int_{D3} C_0\, F_{\mu\nu}{}^{*}F^{\mu\nu} + \cdots \ . \tag{5.8}$$

Promoting these expressions to the non-abelian case, which results in an extra factor of 1/2 due to the normalization of the colour trace, and comparing with eq. (5.4), we can read off that the gauge kinetic function for the fractional D3-brane of type A is

$$f_A(M) = \frac{\tau}{4}\,, \tag{5.9}$$

[15] This extends to the case of the fractional brane of type 0 by interpreting it as a D5-wrapped on $\sum_{A=1}^{3} \hat{e}^A$ with negative orientation and with a suitable magnetic flux turned on.

with

$$\tau \equiv C_0 + \mathrm{i}\mathrm{e}^{-\varphi} \tag{5.10}$$

the axion-dilaton field. Combining eq. (5.9) with eqs. (5.3) and (5.10) leads to

$$g_{\rm YM}^2 = 16\pi \mathrm{e}^{\varphi} \quad \text{and} \quad \theta_{\rm YM} = \frac{\pi C_0}{2} \ . \tag{5.11}$$

The way to arrange the remaining untwisted and twisted scalars as the complex bosons of suitable chiral multiplets is suggested, as remarked above, by the gauge kinetic functions of other D-branes maintaining the same $\mathcal{N} = 1$ supersymmetry selected by the fractional D3-branes. For the untwisted scalars, we can just consider "regular" branes, such as D7-ones wrapped on one of the untwisted 4-cycles. Starting from the wrapped D7-brane DBI-WZ action, in the end one finds (see for instance ref. [2]) that the gauge kinetic function for these branes is

$$f_i(M) = t^i \quad \text{with} \quad t^i \equiv \tilde{c}^i + \frac{\mathrm{i}}{2}|\epsilon^{ijk}|v_j v_k \tag{5.12}$$

where v_i and $\tilde{c}^i$ have been defined at the beginning of this section. The complex fields t^i represent the correct (untwisted) Kähler coordinates to be used for the $\mathcal{N} = 1$ supergravity associated to CY compactifications with D3/D7-branes and O3/O7-planes, together with the τ variable defined in eq. (5.10). Notice also that the imaginary parts of the coordinates (5.12) are related to the volume $\mathcal{V}$ of the $\mathcal{T}_6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, measured in the Einstein frame; in fact

$$\Big(\mathrm{Im}\, t^1 \,\mathrm{Im}\, t^2 \,\mathrm{Im}\, t^3\Big)^{\frac{1}{2}} = v_1 v_2 v_3 = \mathcal{V} \ . \tag{5.13}$$

Let us now return to the gauge theory defined on the fractional D3-branes and on its gauge kinetic function $f_A = \tau/4$. The modulus τ is connected by the residual supercharges to other closed string states and it can be promoted to a chiral superfield $\tau(\theta)$. The complete Lagrangian of the fractional D3-branes, given in eq. (5.5), contains then also the coupling of the gaugino Λ^α to the auxiliary component F^τ of $\tau(\theta)$, namely

$$-\frac{\mathrm{i}}{8\pi}\frac{F^\tau}{4}\,\mathrm{Tr}\left(\Lambda^\alpha \Lambda_\alpha\right) + \text{c.c.}\,, \tag{5.14}$$

which corresponds to the following mass

$$m_\Lambda = \frac{1}{2\mathrm{Im}\, f_A}\frac{F^\tau}{4} = \frac{\mathrm{e}^{\varphi}\, F^\tau}{2} \tag{5.15}$$

for canonically normalized gaugino fields.

The bilinear term (5.14) must coincide with the flux-induced coupling we have computed in section 3.4. In fact, in presence of a G-flux the gauginos acquire mass terms given by eq. (3.44) which must be adapted to our $\mathcal{N} = 1$ orbifold model. This is easily done by taking only the invariant gaugino $\Lambda^0 \equiv \Lambda$. Using appendix A.1 (and in particular eq. (A.22)) we find that the only component of the $G_{mnp}^{\rm IASD}$ tensor which contributes to eq. (3.44) when $A = B = 0$, is its $(3, 0)$ part; thus, after combining eqs. (3.48) and (5.11), we find that the flux-induced gaugino mass term for fractional D3-branes reads

$$-\frac{\mathrm{i}}{2}\,\mathrm{e}^{-\varphi}\,(2\pi\alpha')^{-\frac{1}{2}}\mathcal{N}_F\, G_{(3,0)}\,\mathrm{Tr}\left(\Lambda^\alpha\Lambda_\alpha\right) + \text{c.c.}\ . \tag{5.16}$$

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Comparing with eq. (5.14), we finally deduce that

$$F^{\tau} = 16\pi\, \mathrm{e}^{-\varphi}\, (2\pi\alpha')^{-\frac{1}{2}} \mathcal{N}_F\, G_{(3,0)} \ . \tag{5.17}$$

Later in this section, we will fix the normalization $\mathcal{N}_F$ of the flux vertex by requiring that this expression for F^{τ} matches the one obtained by constructing the bulk low energy Lagrangian.

Comparison with the bulk theory. It is well known (see for example refs. [11, 68]) that the bulk theory for our toroidal compactification yields, after a dimensional reduction to four dimensions and a Weyl rescaling to the $d = 4$ Einstein frame, a $\mathcal{N} = 1$ supergravity theory coupled to vector and matter multiplets in the standard form. This effective theory is therefore specified, besides the gauge kinetic function for the bulk vector multiplets, by the Kähler potential K and by the holomorphic superpotential W for the chiral multiplets. To simplify the treatment, in the following we consider as dynamical only a subset of the compactification moduli; in particular we keep the dependence on the universal chiral multiplet τ of eq. (5.10), but restrict to a slice of the Kähler moduli space in which an overall scale

$$t \equiv t^1 = t^2 = t^3 \tag{5.18}$$

is considered. Such a scale is related to the compactification volume by $\mathcal{V} = (\mathrm{Im}\, t)^{3/2}$ as it follows from eq. (5.13). We also freeze the complex structure moduli u^i to their "trivial" value corresponding to $\mathcal{T}_6$ being the product of three upright tori, *i.e.* we set $u^1 = u^2 = u^3 = \mathrm{i}$; furthermore we neglect the dependence on all the remaining twisted and untwisted moduli.

With these assumptions, the Kähler potential for the bulk theory is

$$K = -\log(\mathrm{Im}\, \tau) - 3\log(\mathrm{Im}\, t) \ . \tag{5.19}$$

When internal 3-form fluxes are turned on, a non-trivial bulk superpotential appears [7, 8] and its expression is

$$W = \frac{1}{\kappa_{10}^2} \int G \wedge \Omega = \frac{4}{\kappa_4^2} G_{(0,3)} \tag{5.20}$$

where Ω is the holomorphic 3-form of the internal space, and κ_{10} and κ_4 are, respectively, the gravitational constants in ten and four dimensions.[16] In eq. (5.20) the 3-form flux is

$$G = F - \tau\, H \tag{5.21}$$

which is the natural extension of eq. (3.20) when g_s is promoted to e^{φ} and the presence of a non-vanishing axion C_0 is taken into account. Note that W has the correct dimensions of (length)$^{-3}$, since κ_4 is a length and the flux is a mass, and that only the ISD component $G_{(0,3)}$ of G is responsible for a non-vanishing W.

[16] In our case the holomorphic 3-form is simply given by $\Omega = dZ^1 \wedge dZ^2 \wedge dZ^3$. In our conventions, we have $\int \bar{\Omega} \wedge \Omega = (2\pi\sqrt{\alpha'})^6$, while $\kappa_{10}^2/\kappa_4^2 = (2\pi\sqrt{\alpha'})^6/4$, where the factor of 1/4 represents the order of the orbifold group.

In presence of a superpotential W, the auxiliary fields in the chiral multiplets are given by the standard supergravity expressions which in our case become

$$
\begin{aligned}
\overline{F}^{\bar\tau} &= -\mathrm{i}\,\kappa_4^2\,\mathrm{e}^{K/2}\,K^{\bar\tau\tau}\,D_\tau W \;=\; 8\,\frac{\mathrm{e}^{-\varphi/2}}{\mathcal{V}}\,\overline{G}_{(0,3)} \quad\Rightarrow\quad F^\tau \;=\; 8\,\frac{\mathrm{e}^{-\varphi/2}}{\mathcal{V}}\,G_{(3,0)}\,,\\
\overline{F}^{\bar t} &= -\mathrm{i}\,\kappa_4^2\,\mathrm{e}^{K/2}\,K^{\bar t t}\,D_t W \;=\; 8\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}^{\frac13}}\,G_{(0,3)} \quad\Rightarrow\quad F^t \;=\; 8\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}^{\frac13}}\,\overline{G}_{(3,0)}\,,
\end{aligned}
\tag{5.22}
$$

where $\overline{G}$ is the complex conjugate of G, $K^{\bar\tau\tau}$ and $K^{\bar t t}$ are the inverse Kähler metrics for τ and t respectively, and the Kähler covariant derivatives of the superpotential are defined as $D_i W = \partial_i W + \big(\partial_i K\big) W$. Thus, by comparing the expression of F^τ derived from the flux-induced gaugino mass and given in eq. (5.17) with eq. (5.22), we find perfect agreement in the structure and can fix the normalization of the closed string vertex for the flux to be

$$
\mathcal{N}_F = \frac{\mathrm{e}^{\varphi/2}}{2\pi\mathcal{V}}\,(2\pi\alpha')^{\frac12}\ . \tag{5.23}
$$

From eq. (3.5) we also infer that (promoting g_s to e^{φ})

$$
\mathcal{N}_H = \frac{\mathrm{e}^{-\varphi/2}}{2\pi\mathcal{V}}\,(2\pi\alpha')^{\frac12}\ . \tag{5.24}
$$

With these normalizations, the closed string vertices (2.20) and (2.23) can be used to derive directly from string amplitudes the terms in the four dimensional effective Lagrangian in the Einstein frame, and the resulting expressions do indeed have the correct normalization that follows from the dimensional reduction of the original Type IIB action in ten dimensions. In this perspective, we point out that the scalar potential due to the chiral multiplets, which has the form

$$
\begin{aligned}
V_F &= \kappa_4^2\,\mathrm{e}^K\,\Big(K^{\tau\bar\tau}D_\tau W D_{\bar\tau}\bar W + K^{t\bar t}D_t W D_{\bar t}\overline{W} - 3\,|W|^2\Big)\\
&= \frac{16}{\kappa_4^2}\,\frac{\mathrm{e}^{\varphi}}{\mathcal{V}^2}\,G_{(3,0)}\,\overline{G}_{(0,3)} = \frac{1}{\kappa_4^2}\,\Big|4\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}}\,G_{(3,0)}\Big|^2\,,
\end{aligned}
\tag{5.25}
$$

coincides with the kinetic terms for the R-R and NS-NS 3-forms in the ten dimensional Einstein frame, given in eq. (3.4), after dimensional reduction to $d = 4$ and rescaling to the four dimensional Einstein frame, if only the (3,0) and (0,3) components of the fluxes are turned on.

Let us also recall that the last term of V_F in the first line of eq. (5.25) is a purely "gravitational" contribution, related to the gravitino mass

$$
\big|m_{3/2}\big| = \kappa_4^2\,\mathrm{e}^{K/2}\,\big|W\big| = \Big|4\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}}\,G_{(0,3)}\Big|\ . \tag{5.26}
$$

From these equations, we see clearly the very different rôle of the ISD flux $G_{(0,3)}$, which induces a gravitino mass, with respect to the IASD flux $G_{(3,0)}$, which is instead responsible for the gaugino mass term. The latter is described by eq.s (5.14) and (5.22) which correspond, according to eq. (5.15), to a canonical gaugino mass

$$
\big|m_\Lambda\big| = \Big|4\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}}\,G_{(3,0)}\Big|\ . \tag{5.27}
$$

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These very well-known results [10−12] will be generalized and extended to instantonic branes in the following section, and the effects on the instanton moduli space of a flux-induced mass term for the gaugino or the gravitino will be determined. We conclude this section by mentioning that the same analysis we have described for fractional D3 branes can be performed without any difficulty in the case of fractional D9 branes. Some details on this are provided in appendix A.3.

6. The rôle of fluxes on fractional D-instantons

In sections 3 and 4 we have computed the fermionic bilinear couplings of the NS-NS and R-R bulk fluxes to open strings with at least one end-point on the D-instanton. The results (3.49) and (4.14) describe deformations of the instanton moduli space of the $\mathcal{N}=4$ gauge theory living on the D3-brane. We now discuss the meaning of these interaction terms in a simple example within the context of the $\mathcal{N}=1$ orbifold compactification introduced in the last section.

Consider a specific node A of the quiver diagram represented in figure 2 and put on it N fractional D3 branes and one fractional D-instanton. The latter describes the $k=1$ gauge instanton for the $\mathcal{N}=1$ $\mathrm{U}(N)$ Yang-Mills theory defined on the world-volume of the space-filling D3-branes. The open strings with at least one end point on it account for the instanton zero-modes in the ADHM construction [17−20]. Specifically, for the D(−1)/D(−1) strings, we have the four bosonic zero-modes x^μ and three auxiliary zero-modes D_c from the NS sector plus two chiral zero-modes θ^α and two anti-chiral zero-modes $\lambda_{\dot\alpha}$ from the R sector. Besides these neutral modes, there are also charged zero-modes from the D3/D(−1) and D(−1)/D3 strings comprising the bosons $w_{\dot\alpha}^{\ u}$ and $\bar w_{\dot\alpha u}$ from the NS sector, and the scalar fermions μ^u and $\bar\mu_u$ from the R sector, where the upper (or lower) index u belongs to the fundamental (or anti-fundamental) representation of $\mathrm{U}(N)$.

The action of the $\mathcal{N}=1$ fractional D-instanton zero-modes turns out to be (see e.g. ref. [25])

$$S_{\rm inst} = 2\pi\mathrm{i}\, f_A + \mathrm{i}\,\lambda_{\dot\alpha}\big(\bar\mu_u w^{\dot\alpha u} + \bar w^{\dot\alpha}_{\ u}\mu^u\big) - \mathrm{i}D_c\,\bar w_{\dot\alpha u}(\tau^c)^{\dot\alpha}_{\ \dot\beta} w^{\dot\beta u} \tag{6.1}$$

where $f_A=\tau/4$ is the gauge kinetic function (5.9) and τ^c are the three Pauli matrices. Note that neither x^μ nor θ^α appear in $S_{\mathrm{D}(-1)}$; in fact they are the Goldstone modes of the supertranslation symmetries broken by the instanton and as such can be identified with the superspace coordinates of the $\mathcal{N}=1$ theory. On the other hand $\lambda_{\dot\alpha}$ and D_c appear only linearly in $S_{\mathrm{D}(-1)}$: they are Lagrange multipliers enforcing the so-called super ADHM constraints. The action (6.1) can be easily derived by computing (mixed) disk amplitudes with insertions of vertex operators representing the various zero-modes [20].

Let us focus in particular on the fermionic moduli. The neutral zero-modes θ^α and $\lambda_{\dot\alpha}$ are clearly described by the chiral and anti-chiral components of the D(−1)/D(−1) fermion that is invariant under the orbifold action (*i.e.* with an internal spinor index 0), namely

$$\Theta^{\alpha 0} \sim g_0\,\theta^\alpha \quad \text{and} \quad \Theta_{\dot\alpha 0} \sim \lambda_{\dot\alpha}\ . \tag{6.2}$$

The extra power of the D(−1) gauge coupling $g_0 = \frac{1}{\sqrt{\pi}}(2\pi\alpha')^{-1}\mathrm{e}^{\varphi/2}$ accounts for the correct scaling dimensions that allow to interpret θ^α as the fermionic superspace coordinate with dimensions of (length)$^{1/2}$. On the other hand, as mentioned above, $\lambda_{\dot\alpha}$ is the Lagrange multiplier for the fermionic ADHM constraint and carries dimensions of (length)$^{-3/2}$, so that no rescaling is needed.

In the charged sector the fermionic moduli μ^u and $\bar\mu_u$ correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ invariant fermions of the strings stretching between the D3-branes and the D-instanton so that, using the notation of section 4, for each colour we have

$$\mu^0 \sim g_0\,\mu \quad \text{and} \quad \bar\mu^0 \sim g_0\,\bar\mu\ . \tag{6.3}$$

As before, an extra power of g_0 is included to account for the correct (length)$^{1/2}$ dimensions of the charged moduli $\mu, \bar\mu$. The normalizations[17] of the fermionic string vertices can then be written as [20]

$$\mathcal{N}_\lambda = (2\pi\alpha')^{\frac{3}{4}}\,, \quad \mathcal{N}_\theta = 4\sqrt{\pi}\,\mathrm{e}^{-\varphi/2}\,\frac{g_0}{\sqrt{2}}\,(2\pi\alpha')^{\frac{3}{4}} \quad \mathcal{N}_\mu = \mathcal{N}_{\bar\mu} = \frac{g_0}{\sqrt{2}}(2\pi\alpha')^{\frac{3}{4}}\ . \tag{6.4}$$

We are now ready to study how the bulk R-R and NS-NS fluxes modify the moduli action. Actually in section 3 we have already computed the flux interactions with the untwisted fermions of a D(−1)-brane (see eq. (3.49)) while in section 4 we computed the flux couplings to the twisted fermions of the D3/D(−1) system (see eq. 4.14). So what we have to do now is simply to insert in these equations the appropriate normalizations discussed above and take into account the identifications of the fluxes with the bulk chiral multiplets explained in the previous section. The flux induced terms in the instanton moduli action are thus[18]

$$S_{\rm inst}^{\rm flux} = -\mathcal{A}^{\rm flux}_{\rm D(-1)} - \mathcal{A}^{\rm flux}_{\rm D3/D(-1)} \tag{6.5}$$

where $\mathcal{A}^{\rm flux}_{\rm D(-1)}$ and $\mathcal{A}^{\rm flux}_{\rm D3/D(-1)}$ are the $A = B = 0$ parts of the amplitudes (3.49) and (4.14), i.e.

$$\begin{aligned} \mathcal{A}^{\rm flux}_{\rm D(-1)} &= -2\pi\mathrm{i}\,c_F(\theta)\,\theta^\alpha\theta_\alpha\,G_{(3,0)} + 2\pi\mathrm{i}\,c_F(\lambda)\,\lambda_{\dot\alpha}\lambda^{\dot\alpha}\,G_{(0,3)}\,,\\ \mathcal{A}^{\rm flux}_{\rm D3/D(-1)} &= -4\pi\mathrm{i}\,c_F(\mu)\,\bar\mu_u\,\mu^u\,G_{(3,0)}\ . \end{aligned} \tag{6.6}$$

In these expressions we have distinguished the c_F coefficients for the various terms to account for the appropriate normalizations of the moduli as discussed before. Recalling that the normalization $\mathcal{C}_{(0)}$ of the disk amplitudes is given by eq. (3.50) with $g^2_{\rm YM}$ defined in (5.11), and that the G fluxes are normalized as indicated in eq. (5.23), we find

$$\begin{aligned} c_F(\theta) &= \mathcal{C}_{(0)}\,\mathcal{N}_F\,\mathcal{N}_\theta^2 = 2\,\frac{\mathrm{e}^{-\varphi/2}}{\mathcal{V}}\,,\\ c_F(\lambda) &= \mathcal{C}_{(0)}\,\mathcal{N}_F\,\mathcal{N}_\lambda^2 = (2\pi\alpha')^2\,\frac{\mathrm{e}^{-\varphi/2}}{4\mathcal{V}}\,,\\ c_F(\mu) &= \mathcal{C}_{(0)}\,\mathcal{N}_F\,\mathcal{N}_\mu^2 = \frac{\mathrm{e}^{\varphi/2}}{8\pi\mathcal{V}}\ . \end{aligned} \tag{6.7}$$

[17]The extra factor of $4\sqrt{\pi}\,\mathrm{e}^{-\varphi/2}$ in the definition of $\mathcal{N}_\theta$ with respect to [20] is needed, as we will see, in order to identify θ with the superspace coordinates.

[18]Recall that in Euclidean spaces there is a minus sign in going from an amplitude to an action.

Notice that all factors of α' cancel in $c_F(\theta)$ and $c_F(\mu)$, but they survive in $c_F(\lambda)$ whose scaling dimension of (length)^4 is the correct one for the λ^2 term of $\mathcal{A}^{\rm flux}_{\rm D(-1)}$ in (6.6) to be dimensionless. Using these coefficients in (6.6) and exploiting the results of the previous section (in particular eqs. (5.27) and (5.26)), we can rewrite the flux-induced moduli action as follows

$$\begin{aligned} S^{\rm flux}_{\rm inst} &= 4\pi {\rm i} \frac{{\rm e}^{-\varphi/2} G_{(3,0)}}{\mathcal{V}}\, \theta^\alpha \theta_\alpha \,-\, {\rm i}\pi\, (2\pi\alpha')^2\, \frac{{\rm e}^{-\varphi/2} G_{(0,3)}}{2\mathcal{V}}\, \lambda_{\dot\alpha}\lambda^{\dot\alpha} \,+\, {\rm i}\, \frac{G_{(3,0)}}{2\mathcal{V}}\, \bar\mu_u \mu^u \\ &= \frac{{\rm i}\pi}{2}\, F^\tau\, \theta^\alpha\theta_\alpha \,-\, \frac{{\rm i}\pi}{8}\, \kappa_4^2\, (2\pi\alpha')^2\, {\rm e}^{-\varphi}\, {\rm e}^{K/2}\, \lambda_{\dot\alpha}\lambda^{\dot\alpha} \,+\, \frac{{\rm i}\,{\rm e}^{\varphi}}{16} F^\tau\, \bar\mu_u \mu^u\ . \end{aligned} \tag{6.8}$$

The θ^2 term represents the auxiliary component of the gauge kinetic function $f_A = \tau/4$, which is therefore promoted to the full chiral superfield $f_A(\theta) = \tau(\theta)/4$ in complete (and expected) analogy with what happened on the D3-branes. The other two terms are less obvious: they represent the explicit effects of a background G flux on the instanton moduli space, and are the strict analogue for the instanton action of the soft supersymmetry breaking terms of the gauge theory. In particular the $\bar\mu\mu$ term is related to the IASD flux component $G_{(3,0)}$ which is responsible for the gaugino mass m_Λ, while the λ^2 term represents a truly stringy effect on the instanton moduli space and is related to the ISD flux component $G_{(0,3)}$ which gives rise to the gravitino mass $m_{3/2}$.

The study of these terms, of their consequences for the instanton calculus and of the non-perturbative effects that they may induce in the gauge theory will be presented in a companion paper [69]. Here we simply mention that the above analysis can be easily generalized to SQCD models with flavored matter in the fundamental (anti-fundamental) representation and also to configurations in which the fractional D-instanton occupies a node of the quiver diagram which is *not* occupied by the colored or flavored space-filling branes. For these "exotic" instanton configurations there are no bosonic moduli $w_{\dot\alpha}$ and $\bar w_{\dot\alpha}$ and the action (6.1) simply reduces to first term involving the gauge kinetic function. Since the neutral anti-chiral fermionic moduli $\lambda_{\dot\alpha}$ do not couple to anything, to avoid a trivial vanishing result upon integration over the moduli space, it is necessary to remove them or to lift them. As we have explicitly seen, by coupling the fractional D-instanton to an ISD G-flux of type (0,3) it is possible to achieve this goal exploiting the λ^2 term proportional to the gravitino mass.

Finally, we observe that using the explicit expression (3.29) of the fermionic coupling, the flux-induced moduli amplitude on a E3 instanton $\mathcal{A}^{\rm flux}_{\rm E3}$ contains terms of the form

$$\theta^\alpha\theta_\alpha\, \bar G_{(3,0)} \quad \text{and} \quad \lambda_{\dot\alpha}\lambda^{\dot\alpha}\, \bar G_{(0,3)} \tag{6.9}$$

where $\bar G_{(3,0)}$ and $\bar G_{(0,3)}$ are, respectively, the $(3,0)$ and $(0,3)$ components of $\bar G = F + \frac{\rm i}{g_s} H$. Thus, on a E3 instanton a G-flux of type (2,1) or (0,3) cannot lift the $\lambda_{\dot\alpha}$'s since its conjugate flux $\bar G$ does not contain a $(0,3)$-component, in full agreement with the findings of ref. [36]. However, as is clear from (6.9), a G-flux of type (3,0) can lift the anti-chiral zero-modes $\lambda_{\dot\alpha}$.

7. Summary of results

In this paper we have computed the couplings of NS-NS and R-R fluxes to fermionic bilinears living on general brane intersections (including instantonic ones). The couplings have been extracted from disk amplitudes among two open string vertex operators and one closed string vertex representing the background fluxes. The results for the R-R and NS-NS amplitudes are given in eqs. (2.43) and (2.51). At leading order in α' they describe fermionic mass terms induced at linear order in the R-R and NS-NS fluxes for open string modes with boundary conditions encodes in the magnetized reflection matrices $R_0, \mathcal{R}_0$ and the open string twists $\vec{\vartheta}$.

The case $\vec{\vartheta} = 0$ corresponds to open strings starting and ending on two parallel D-branes. The result in this case can be written in the simple form

$$\mathcal{A} = -\frac{2\pi \mathrm{i}}{3!}\, c_F\, \Theta\Gamma^{MNP}\Theta\, T_{MNP} \tag{7.1}$$

where c_F is a normalization factor and

$$T_{MNP} = \left(F\mathcal{R}_0\right)_{MNP} + \frac{3}{g_s}\left(\partial B R_0\right)_{[MNP]} \,. \tag{7.2}$$

This formula shows that different branes couple to different combinations of the R-R and NS-NS fields. For compactifications to $d = 4$ in presence of 3-form internal fluxes the explicit form of the T tensors are displayed in table 1 for spacetime filling D-branes and in table 2 for instantonic branes. For spacetime filling branes, the T-tensor describes the structure of soft fermionic mass terms for a general D-brane intersection. For Euclidean branes, they accounts for fermionic mass terms in the instanton moduli space action modifying the fermionic zero mode structure of the instanton. Our results are in perfect agreement with those of refs. [10, 12, 48, 50 − 53] that have been derived with pure supergravity methods and generalize them to generic (instantonic or not) D-brane intersections. The effects of open string magnetic fluxes can be easily incorporated into these formulas via the reflection matrices $R_0(\mathcal{F})$ and $\mathcal{R}_0(\mathcal{F})$. As an example, the explicit form of the T-tensor for Euclidean magnetized E5-branes has been given in eq. (3.36).

The cases of open strings ending on D3-branes and D-instantons have been studied in detail. For D3-branes in flat space one obtains

$$\mathcal{A}_{\mathrm{D3}} = \frac{2\pi \mathrm{i}}{3!}\, c_F(\Lambda)\, \mathrm{Tr}\Big[\, \Lambda^{\alpha A}\Lambda_{\alpha}{}^{B}\left(\overline{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp} - \bar{\Lambda}_{\dot{\alpha}A}\bar{\Lambda}^{\dot{\alpha}}{}_{B}\left(\Sigma^{mnp}\right)^{AB}\left(G^{\mathrm{IASD}}_{mnp}\right)^{*}\Big] \tag{7.3}$$

with $G = F - \tau H$, G^{IASD} its imaginary anti-self-dual part and $c_F(\Lambda)$ a normalization factor. This formula encodes the structure of soft symmetry breaking terms in $\mathcal{N} = 4$ gauge theory induced by NS-NS and R-R fluxes.

The coupling of fluxes to D-instantons is given instead by

$$\mathcal{A}_{\mathrm{D}(-1)} = \frac{2\pi \mathrm{i}}{3!}\Big[\, c_F(\theta)\, \theta^{\alpha A}\theta_{\alpha}{}^{B}\left(\overline{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp} + c_F(\lambda)\, \lambda_{\dot{\alpha}A}\lambda^{\dot{\alpha}}{}_{B}\left(\Sigma^{mnp}\right)^{AB} G^{\mathrm{ISD}}_{mnp}\Big] \,. \tag{7.4}$$

The case $\vec{\vartheta} \neq 0$ describes the couplings of open strings stretching between non-parallel stacks of D-branes. For spacetime filling D-branes the corresponding open string excitations describe chiral matter transforming in bi-fundamental representations of the gauge group and always contain massless chiral fermions. The case, where open strings are twisted by

$\vartheta = \frac{1}{2}$ along the spacetime directions, describes the charged moduli of gauge or exotic instantons. For gauge instantons in $\mathcal{N}=4$ gauge theory one finds the flux induced action

$$\mathcal{A}_{\mathrm{D3/D(-1)}} = \frac{4\pi \mathrm{i}}{3!}\, c_F(\mu)\, \bar{\mu}^A \mu^B \left(\overline{\Sigma}^{mnp}\right)_{AB} G^{\mathrm{IASD}}_{mnp} \ . \tag{7.5}$$

The results obtained here extend straightforwardly to less supersymmetric theories and to exotic instantons. In particular for pure $\mathcal{N}=1$ SYM, the flux couplings for both gauge and exotic instantons follow from (7.3), (7.4), (7.5) by restricting the spinor components to $A = B = 0$. The only contributions to fermionic mass terms come in this case from the components $G_{(3,0)}$ and $G_{(0,3)}$ related to the soft symmmetry breaking gaugino and gravitino masses. Explicitly for $\mathcal{T}_6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ we have

$$\begin{aligned} \left|m_\Lambda\right| &= \left|\frac{\kappa_4^2}{2}\,\mathrm{e}^{\varphi}\,\mathrm{e}^{K/2}\,D^\tau W\right| = \left|4\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}}\,G_{(3,0)}\right| ,\\ \left|m_{3/2}\right| &= \left|\kappa_4^2\,\mathrm{e}^{K/2}\,W\right| = \left|4\,\frac{\mathrm{e}^{\varphi/2}}{\mathcal{V}}\,G_{(0,3)}\right| , \end{aligned} \tag{7.6}$$

and the fermionic flux couplings can be written as

$$\begin{aligned} \mathcal{A}_{\mathrm{D3}} &= -\frac{\mathrm{i}}{16\pi}\, m_\Lambda\, \mathrm{e}^{-\varphi}\, \mathrm{Tr}\big[\,\Lambda^\alpha \Lambda_\alpha\big] + \mathrm{c.c.}\,,\\ \mathcal{A}_{\mathrm{D(-1)}} &= -\pi \mathrm{i}\, m_\Lambda\, \mathrm{e}^{-\varphi}\,\theta^\alpha\theta_\alpha + \frac{\pi \mathrm{i}}{8}\,(2\pi\alpha')^2\, m_{3/2}\,\mathrm{e}^{-\varphi}\,\lambda_{\dot\alpha}\lambda^{\dot\alpha}\,,\\ \mathcal{A}_{\mathrm{D3/D(-1)}} &= -\frac{\mathrm{i}}{8}\, m_\Lambda\,\bar{\mu}_u \mu^u \ . \end{aligned} \tag{7.7}$$

These flux couplings modify the zero mode structure of the instanton and allow for new low energy coupling in the D3-brane action. In particular, we observe that the presence of λ-fermionic zero modes typically signals an obstruction to the generation of non-perturbative superpotentials via exotic instantons. This difficulty can be overcome by the introduction of an O-plane leading to O(1)-instantons with no λ-modes. The presence of the λ^2-term in (7.7) suggests that R-R and NS-NS fluxes can provide a valid alternative mechanism in the case of oriented gauge theories. A precise study of the low energy couplings generated by instantons in presence of such fluxes will be presented in the companion paper [69].

Acknowledgments

We thank C. Bachas, P. Di Vecchia, D. Duò, R. Marotta, I. Pesando, R. Russo and M. Serone for many useful discussions. This work is partially supported by the European Commission FP6 Programme under contracts MRTN-CT-2004-005104 (in which A.L. is associated to the University of Torino), MRTN-CT-2004-512194 and MRTNCT-2004-503369 and by the NATO grant PST.CLG.978785. L.F. would like to thank C. Bachas for kind support and LPTENS for wonderful hospitality.

A. Technical details

A.1 Notations and conventions

We use the following notations for space-time indices in the real basis:

- $d = 10$ vector indices: $M, N, \ldots \in \{0, \ldots, 9\}$;
- $d = 4$ vector indices: $\mu, \nu, \ldots \in \{0, \ldots, 3\}$;
- $d = 6$ vector indices: $m, n, \ldots \in \{4, \ldots, 9\}$.

The corresponding complex indices are denoted by $I, J = 1, \ldots, 5$ with $I = i = 1, 2, 3$ referring to the coordinates of the six-dimensional space and $I = 4, 5$ referring to the four-dimensional space-time directions. Even with Euclidean signatures we use the real index 0.

Γ-matrices in ten dimensions. In a $d = 10$ Euclidean space the Γ matrices which satisfy $\left\{\Gamma^M, \Gamma^N\right\} = 2\delta^{MN}$, can be given the following explicit representation in terms of the Pauli matrices τ^c:

$$\begin{aligned}
\Gamma^0 &= \tau^1 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \\
\Gamma^1 &= \tau^2 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \\
\Gamma^2 &= \tau^3 \otimes \tau^1 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \\
\Gamma^3 &= \tau^3 \otimes \tau^2 \otimes \mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} \\
&\vdots \\
\Gamma^8 &= \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^1 \\
\Gamma^9 &= \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^2
\end{aligned} \tag{A.1}$$

The charge conjugation matrix C satisfies

$$C\Gamma^M C^{-1} = -(\Gamma^M)^t \quad \text{with} \quad C^t = -C\,, \tag{A.2}$$

and in the above representation is

$$C = \tau^2 \otimes \tau^1 \otimes \tau^2 \otimes \tau^1 \otimes \tau^2 \ . \tag{A.3}$$

The charge conjugation matrix is used to raise and lower the 32-dimensional spinor indices $(\widehat{\mathcal{A}}, \widehat{\mathcal{B}}, \ldots)$ of the Γ-matrices according to

$$(\Gamma^M)^{\widehat{\mathcal{A}}\widehat{\mathcal{B}}} \equiv (\Gamma^M)^{\widehat{\mathcal{A}}}{}_{\widehat{\mathcal{C}}}\,(C^{-1})^{\widehat{\mathcal{C}}\widehat{\mathcal{B}}} \quad \text{and} \quad (\Gamma^M)_{\widehat{\mathcal{A}}\widehat{\mathcal{B}}} \equiv (C)_{\widehat{\mathcal{A}}\widehat{\mathcal{C}}}\,(\Gamma^M)^{\widehat{\mathcal{C}}}{}_{\widehat{\mathcal{B}}} \ . \tag{A.4}$$

The chirality matrix is defined by

$$\Gamma^{\mathrm{E}}_{(11)} = -\mathrm{i}\,\Gamma^0\Gamma^1 \ldots \Gamma^9 = \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^3 \otimes \tau^3 \ . \tag{A.5}$$

The above expressions are useful to obtain the factorization of the $d = 10$ matrices when the ten-dimensional space is split into $4 + 6$. In fact, by writing

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1} \ , \quad \Gamma^m = \gamma_{(5)} \otimes \gamma^m \ , \quad \Gamma^{\mathrm{E}}_{(11)} = \gamma_{(5)} \otimes \gamma_{(7)} \ , \quad C = C_4 \otimes C_6\,, \tag{A.6}$$

we can read off the explicit representation of the Dirac matrices γ^μ and γ^m for $d = 4$ and $d = 6$, respectively, of the corresponding chirality matrices $\gamma_{(5)}$ and $\gamma_{(7)}$, and of the charge conjugation matrices C_4 and C_6.

$2\,\vec{\epsilon}$	chirality
$(++)$	$+$
$(-+)$	$-$
$(+-)$	$-$
$(--)$	$+$

Table 6: Ordering of spinor indices in four dimensions.

Γ-matrices in four dimensions. The $d = 4$ matrices γ^μ which can be read from eqs. (A.1) and (A.6), are

$$\gamma^0 = \tau^1 \otimes \mathbf{1}\,, \quad \gamma^1 = \tau^2 \otimes \mathbf{1}\,, \quad \gamma^2 = \tau^3 \otimes \tau^1\,, \quad \gamma^3 = \tau^3 \otimes \tau^2\,, \tag{A.7}$$

while the chirality and charge conjugation matrices are

$$\gamma_{(5)} = -\,\gamma^0\gamma^1\gamma^2\gamma^3 = \tau^3 \otimes \tau^3 \quad \text{and} \quad C_4 = \tau^2 \otimes \tau^1\ . \tag{A.8}$$

In this tensor product basis the four spinor indices are ordered as in table 6. However, it is often useful to rearrange them in order to have first the two chiral indices $\alpha \in \{(++), (--)\}$ and then the two anti-chiral ones $\dot{\alpha} \in \{(-+), (+-)\}$, in such a way that the chirality matrix takes the more conventional form $\gamma_{(5)} = \mathbf{1} \otimes \tau^3$. With such a rearrangement the above Euclidean Dirac matrices γ^μ become

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \tag{A.9}$$

with $\sigma^\mu = \big(\mathbf{1},\, -\mathrm{i}\tau^3,\, \mathrm{i}\tau^2,\, -\mathrm{i}\tau^1\big)$ and $\overline{\sigma}^\mu = \big(\mathbf{1},\, \mathrm{i}\tau^3,\, -\mathrm{i}\tau^2,\, \mathrm{i}\tau^1\big)$. The matrices $\big(\sigma^\mu\big)_{\alpha\dot{\beta}}$ and $\big(\overline{\sigma}^\mu\big)^{\dot{\alpha}\beta}$ act on spinors of definite chirality ψ_α and $\psi^{\dot{\alpha}}$ as

$$\big(\sigma^\mu\big)_{\alpha\dot{\beta}}\,\psi^{\dot{\beta}} \quad \text{and} \quad \big(\overline{\sigma}^\mu\big)^{\dot{\alpha}\beta}\,\psi_\beta\ . \tag{A.10}$$

After the rearrangement of indices, the charge conjugation matrix becomes

$$C_4 = \tau^2 \otimes \tau^3\ ; \tag{A.11}$$

thus it is block diagonal with $\big(C_4\big)^{\alpha\beta} = -\mathrm{i}\,\epsilon^{\alpha\beta}$ and $\big(C_4\big)_{\dot{\alpha}\dot{\beta}} = \mathrm{i}\,\epsilon_{\dot{\alpha}\dot{\beta}}$.

Γ-matrices in six dimensions. The $d = 6$ matrices γ^m which can be read from eqs. (A.1) and (A.6), are

$$\begin{aligned} &\gamma^4 = \tau^1 \otimes \mathbf{1} \otimes \mathbf{1}\,, \quad \gamma^5 = \tau^2 \otimes \mathbf{1} \otimes \mathbf{1}\,, \quad \gamma^6 = \tau^3 \otimes \tau^1 \otimes \mathbf{1} \\ &\gamma^7 = \tau^3 \otimes \tau^2 \otimes \mathbf{1}\,, \quad \gamma^8 = \tau^3 \otimes \tau^3 \otimes \tau^1\,, \quad \gamma^9 = \tau^3 \otimes \tau^3 \otimes \tau^2 \end{aligned} \tag{A.12}$$

while the corresponding chirality and charge conjugation matrices are

$$\gamma_{(7)} = \mathrm{i}\,\gamma^4\gamma^5 \ldots \gamma^9 \;=\; \tau^3 \otimes \tau^3 \otimes \tau^3 \quad \text{and} \quad C_6 = \tau^2 \otimes \tau^1 \otimes \tau^2\ . \tag{A.13}$$

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$2\vec{\epsilon}$	chirality
$(+++)$	$+$
$(-++)$	$-$
$(+-+)$	$-$
$(--+)$	$+$
$(++-)$	$-$
$(-+-)$	$+$
$(+--)$	$+$
$(---)$	$-$

Table 7: Ordering of spinor indices in six dimensions.

In this case the eight spinor indices are ordered according to table 7. but again they can be rearranged in such a way to put first the chiral ones and then the anti-chiral ones, and have the chirality matrix in the standard form $\gamma_{(7)} = \mathbf{1} \otimes \mathbf{1} \otimes \tau^3$. In this basis the matrices $\gamma^m\, C_6^{-1}$ may be written in the block diagonal form

$$\gamma^m\, C_6^{-1} = \begin{pmatrix} \Sigma^m & 0 \\ 0 & \overline{\Sigma}^m \end{pmatrix} \tag{A.14}$$

where $(\Sigma^m)^{AB}$ and $(\overline{\Sigma}^m)_{AB}$ are 4×4 anti-symmetric matrices.

If we order the four chiral indices as $\{(+++), (+--), (-+-), (--+)\}$ and the four anti-chiral indices as $\{(---), (-++), (+-+), (++-)\}$ (see also eq. (9) below), we have

$$\begin{aligned} \Sigma^m &= \left(\eta^3, -\mathrm{i}\overline{\eta}^3, \eta^2, -\mathrm{i}\overline{\eta}^2, \eta^1, \mathrm{i}\overline{\eta}^1\right), \\ \overline{\Sigma}^m &= \left(\eta^3, \mathrm{i}\overline{\eta}^3, -\eta^2, -\mathrm{i}\overline{\eta}^2, \eta^1, -\mathrm{i}\overline{\eta}^1\right), \end{aligned} \tag{A.15}$$

where η^c and $\overline{\eta}^c$ are, respectively, the self-dual and anti-self-dual 't Hooft symbols. Proceeding in a similar way for the antisimmetrized product of three matrices, we find

$$\gamma^{mnp}\, C_6^{-1} = \begin{pmatrix} \Sigma^{mnp} & 0 \\ 0 & \overline{\Sigma}^{mnp} \end{pmatrix}, \tag{A.16}$$

where $(\Sigma^{mnp})^{AB}$ and $(\overline{\Sigma}^{mnp})_{AB}$ the 4×4 symmetric matrices that appear in section 3. Using the properties of the chirality and charge conjugation matrices, it is easy to show the following imaginary self-duality properties

$$*_6\Sigma^{mnp} = -\mathrm{i}\, \Sigma^{mnp}\,, \quad *_6\overline{\Sigma}^{mnp} = +\mathrm{i}\, \overline{\Sigma}^{mnp}\,. \tag{A.17}$$

Useful formulas. The previous formulas allow us to obtain the explicit expressions for the fermion bilinears which have been discussed in sections 2 and 3. In this respect we point out that in writing a fermion bilinear, like the one appearing for instance in eq. (3.9), we always understand the inverse charge conjugation matrix C^{-1}. The precise expression of the bilinear is then

$$\Theta\Gamma^{mnp}\Theta \equiv \Theta_{\mathcal{A}}\, (\Gamma^{mnp}C^{-1})^{\mathcal{AB}}\, \Theta_{\mathcal{B}}\,. \tag{A.18}$$

Using the 4+6 decomposition discussed above, we obtain

$$\Gamma^{mnp}C^{-1} = \left(\gamma_{(5)}C_4^{-1}\right) \otimes \left(\gamma^{mnp}C_6^{-1}\right) = \begin{pmatrix} \tau^2 & 0 \\ 0 & \tau^2 \end{pmatrix} \otimes \begin{pmatrix} \Sigma^{mnp} & 0 \\ 0 & \overline{\Sigma}^{mnp} \end{pmatrix}, \tag{A.19}$$

so that eq. (A.18) can be rewritten as

$$\Theta_{\mathcal{A}}\left(\Gamma^{mnp}C^{-1}\right)^{\mathcal{AB}}\Theta_{\mathcal{B}} = -\mathrm{i}\,\Theta^{\alpha A}\,\epsilon_{\alpha\beta}\Theta^{\beta B}(\overline{\Sigma}^{mnp})_{AB} - \mathrm{i}\,\Theta_{\dot{\alpha}A}\,\epsilon^{\dot{\alpha}\dot{\beta}}\Theta_{\dot{\beta}B}(\Sigma^{mnp})^{AB} \tag{A.20}$$

which coincides with eq. (3.39).

Finally, we observe that the natural ordering, given in table 7, of the spinor indices for the tensor product representation (A.12) is particularly convenient if one uses the complex basis in the internal six-dimensional space. Indeed, computing the holomorphic and anti-holomorphic products γ^{123} and $\gamma^{\bar{1}\bar{2}\bar{3}}$ and combining them with the charge conjugation matrix we find

$$\begin{aligned} \gamma^{123}\,C_6^{-1} &= -\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ \gamma^{\bar{1}\bar{2}\bar{3}}\,C_6^{-1} &= +\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{A.21}$$

from which we immediately see that $\Sigma^{123} = \overline{\Sigma}^{\bar{1}\bar{2}\bar{3}} = 0$ and that the only non-vanishing entries of the matrices $\Sigma^{\bar{1}\bar{2}\bar{3}}$ and $\overline{\Sigma}^{123}$ are, respectively, the upper most left and the lower most right, that is

$$\left(\Sigma^{\bar{1}\bar{2}\bar{3}}\right)^{+++,+++} = 1 \quad \text{and} \quad \left(\overline{\Sigma}^{123}\right)_{---,---} = -1\,. \tag{A.22}$$

A.2 The orbifold $\mathcal{T}_6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ acting on the orthonormal complex coordinates Z^i of $\mathcal{T}_6$ as in table 4 is a discrete subgroup of SO(6) that contains 4 elements h_I ($I = 0, 1, 2, 3$), with $h^0 \equiv e$ being the identity element, and

$$h^1 = \mathrm{e}^{\mathrm{i}\pi(J_3 - J_2)}\,, \qquad h^2 = \mathrm{e}^{\mathrm{i}\pi(J_1 - J_3)}\,, \qquad h^3 \equiv h_1 h_2 = \mathrm{e}^{\mathrm{i}\pi(J_1 - J_2)} \tag{A.23}$$

where $J_{1,2,3}$ are the generators of rotations in the 4-5, 6-7 and 8-9 planes respectively. We may summarize the transformation properties for the conformal fields ∂Z^i and Ψ^i ($i = 1, 2, 3$) in the Neveu-Schwarz sector by means of the following table:

conf. field	irrep
∂Z^i, Ψ^i	R_i

, (A.24)

where $\{R_A\} = \{R_0, R_i\}$ are the irreducible representations of $\mathbb{Z}_2 \times \mathbb{Z}_2$, identified by writing the character table $(\mathrm{Ch})^I_A = \mathrm{tr}_{R_A}\left(h^I\right)$ of the group, given in table 8. The Clebsch-Gordan series for these representations is simply given by

$$R_0 \otimes R_A = R_A\,, \qquad R_i \otimes R_j = \delta_{ij} R_0 + |\epsilon_{ijk}| R_k\,, \tag{A.25}$$

	e	h_1	h_2	h_3
R_0	1	1	1	1
R_1	1	1	-1	-1
R_2	1	-1	1	-1
R_3	1	-1	-1	1

Table 8: Character table of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$.

irrep R_A	S^A	S_A
R_0	$S^0 \equiv S^{+++}$	$S_0 \equiv S_{---}$
R_1	$S^1 \equiv S^{+--}$	$S_1 \equiv S_{-++}$
R_2	$S^2 \equiv S^{-+-}$	$S_2 \equiv S_{+-+}$
R_3	$S^3 \equiv S^{--+}$	$S_3 \equiv S_{++-}$

Table 9: Transformation properties of the spin fields with respect to the orbifold group.

and is crucial in determining the open string spectrum.

Recall that through the bosonization procedure [60] the chiral spin fields $S^A \sim \mathrm{e}^{\mathrm{i}\vec{\epsilon}^A\cdot\vec{\varphi}}$ of SO(6) and the anti-chiral ones $S_A \sim \mathrm{e}^{\mathrm{i}\vec{\epsilon}_A\cdot\vec{\varphi}}$ are associated respectively to the SO(6) spinor weights $\vec{\epsilon}^A = \frac{1}{2}(\pm,\pm,\pm)$ with the product of signs being positive, and $\vec{\epsilon}_A = \frac{1}{2}(\pm,\pm,\pm)$ with the product of signs being negative. Using this information, we easily deduce from (A.23) the transformation properties of the various spin fields, which are summarized in table 9. In other words, we can order the internal spinor indices so that S^A and S_A transform in the irrep R_A.

Closed strings on the orbifold have different sectors. The untwisted sector simply contains the closed string states defined on the covering space $\mathcal{T}_6$ which are invariant under the orbifold action. The twisted sectors are in correspondence with the 16 fixed planes ($a = 1,\ldots,16$) of the action of a nontrivial element h^i. The vertex operators in a twisted sector contain left- and right-moving twist fields $\Delta^i_a(z)$ and $\tilde{\Delta}^i_a(z)$, and must be invariant under the orbifold. If, as explained in the main text, we assume that the orbifold group does not act on the twist fields, it is not difficult to write down all the massless vertices.

Let us also notice that the orbifold projection leaves only two bulk supercharges, whose Weyl components in the $-1/2$ picture are

$$Q_\alpha = \oint \frac{dz}{2\pi\mathrm{i}}\, S_\alpha S_0\, \mathrm{e}^{-\frac{\phi}{2}}(z)\,, \quad Q^{\dot\alpha} = \oint \frac{dz}{2\pi\mathrm{i}}\, S^{\dot\alpha} S^0\, \mathrm{e}^{-\frac{\phi}{2}}(z) \tag{A.26}$$

for the left-moving ones, and

$$\tilde{Q}_\alpha = \oint \frac{d\bar{z}}{2\pi\mathrm{i}}\, \tilde{S}_\alpha \tilde{S}_0\, \mathrm{e}^{-\frac{\tilde{\phi}}{2}}(z)\,, \quad \tilde{Q}^{\dot\alpha} = \oint \frac{d\bar{z}}{2\pi\mathrm{i}}\, \tilde{S}^{\dot\alpha} \tilde{S}^0\, \mathrm{e}^{-\frac{\tilde{\phi}}{2}}(z) \tag{A.27}$$

for the right-moving ones.

A.3 Soft supersymmetry breaking on fractional D9 branes

In the orbifold $\mathcal{T}_6/\mathbb{Z}_2 \times \mathbb{Z}_2$, we can realize an $\mathcal{N}=1$ $d=4$ gauge theory using fractional D9 branes that completely wrap the internal compact space. Such brane configuration preserves a different $\mathcal{N}=1$ supersymmetry with respect to the fractional D3 branes considered in section 5.2, and thus the moduli fields organize into chiral multiplets with respect to this new supersymmetry. The bulk Lagrangian (which is in fact the same since we have not changed the compactification manifold) can be rewritten in terms of these multiplets via a different Kähler potential and superpotential. Again, this allows to relate the flux-induced gaugino mass term to the value of the auxiliary component of the gauge kinetic function.

For the reasons already explained in the case of D3 branes, we are interested mostly in the untwisted couplings, which can be deduced by reducing to four dimensions the usual DBI-WZ action of a D9 brane on $\mathcal{T}_6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ given by

$$-T_9 \int_{D3} \int_{\mathcal{T}_6/(\mathbb{Z}_2\times\mathbb{Z}_2)} \mathrm{e}^{-\varphi} \sqrt{-\det(G_{(10)}+\mathcal{F})} + T_9 \int_{D3} \int_{\mathcal{T}_6/(\mathbb{Z}_2\times\mathbb{Z}_2)} \sum_{n=0}^{5} C_{2n}\, \mathrm{e}^{\mathcal{F}} \tag{A.28}$$

where $T_9 = (4\pi)^{-1}(2\pi\alpha')^{-2}(2\pi\sqrt{\alpha'})^{-6}$. From this expression it follows that the quadratic part in the gauge fields F, after promoting the latter to the non-abelian case and switching to the Einstein frame, is

$$-\frac{\mathrm{e}^{\varphi/2}\mathcal{V}}{32\pi} \int_{D_3} d^4x\, \mathrm{Tr}\big(F_{\mu\nu}F^{\mu\nu}\big) + \frac{\widetilde{C}}{32\pi} \int_{D_3} d^4x\, \mathrm{Tr}\big(F_{\mu\nu}{}^{*}F^{\mu\nu}\big)\,, \tag{A.29}$$

where $\widetilde{C}$ is[19]

$$\int_{\mathcal{T}_6/(\mathbb{Z}_2\times\mathbb{Z}_2)} C_6 = \frac{1}{4}(2\pi\sqrt{\alpha'})^{-6}\, \tilde{C}\ . \tag{A.30}$$

Comparing with eq. (5.4), we see that the untwisted part of the gauge kinetic function $f_A^{(9)}$ for any type A of fractional D9 branes reads

$$f_A^{(9)} = \frac{s}{4} \quad \text{with} \quad s = \tilde{C} + \mathrm{i}\mathrm{e}^{\varphi/2}\mathcal{V}\ . \tag{A.31}$$

In a similar way one can consider D5 branes wrapped on untwisted cycles e^i, which preserve the same supersymmetry of the D9 branes. The way to combine the untwisted moduli into chiral multiplets is again suggested by the gauge kinetic functions $f_i^{(5)}$. Extracting the quadratic terms in the gauge fields from the wrapped D5-brane DBI-WZ action, it is straightforward to obtain

$$f_i^{(5)} = \frac{1}{4} r^i \quad \text{with} \quad r^i \equiv c^i + \mathrm{i}\mathrm{e}^{-\varphi/2}\, v^i\ . \tag{A.32}$$

[19] This pseudoscalar modulus is in fact related, through the duality between F_7 and F_3, to the two-form C_2 with indices in the space-time directions. Notice also that

$$\int_{\mathcal{T}_6/(\mathbb{Z}_2\times\mathbb{Z}_2)} d^6y \sqrt{-g_{(6)}} = \frac{1}{4}\,(2\pi\sqrt{\alpha'})^6\, \mathrm{e}^{3\varphi/2}\mathcal{V}$$

since it corresponds to the internal volume in the string frame, which is related by the factor of $\mathrm{e}^{3\phi/2}$ to the Einstein frame volume $\mathcal{V}$. This explains the prefactor in eq. (A.29).

Notice that c^i and v^i are invariant under the O9 orientifold projection appropriate to our situation.

By supersymmetry, the complex scalars represents the lowest component of a chiral superfield, and the supersymmetric $\mathcal{N}=1$ Lagrangian (5.5), contains the coupling of the gaugino to the auxiliary components of this multiplet. For D9-branes

$$-\frac{\mathrm{i}}{32\pi} F^s \,\mathrm{Tr}\left(\Lambda^\alpha \Lambda_\alpha\right) + \text{c.c.} \tag{A.33}$$

This has to be compared to the gaugino mass term which follows from the D9 flux coupling indicated in table 1. To do so we have to adapt the steps used to arrive at eq. (3.44) in the D3 case, since now the normalization c_F contains contains the topological factor $\mathcal{C}_{(10)}$ suitable for D9 disk amplitude, namely [20]

$$\mathcal{C}_{(10)} = \frac{\mathcal{C}_{(4)}}{(2\pi\sqrt{\alpha'})^6} \ . \tag{A.34}$$

On the other hand to obtain the four dimensional couplings, we have to dimensionally reduce on $\mathcal{T}_6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, gaining a factor of $(2\pi\sqrt{\alpha'})^6\, \mathrm{e}^{3\varphi/2}\, \mathcal{V}$, to obtain the 4d coupling. In the end, we find the term

$$-\frac{\mathrm{i}}{4\pi}\, \mathrm{e}^{\varphi/2}\, \mathcal{V}\, (2\pi\alpha')^{-\frac{1}{2}}\, \mathcal{N}_F\, F_{(3,0)} = -\frac{\mathrm{i}}{4\pi}\, \mathrm{e}^{\varphi}\, F_{(3,0)} \tag{A.35}$$

where in the second step we used the normalization of the flux vertex already fixed in eq. (5.23) so that by comparison with eq. (A.33) the auxiliary field must be given by

$$F^s = 8\, \mathrm{e}^{\varphi}\, F_{(3,0)} \ . \tag{A.36}$$

Analogously to what we did in the D3-brane case, we restrict to the slice of moduli space spanned by s and by the overall scale

$$r \equiv r^1 = r^2 = r^3 \ ; \tag{A.37}$$

this scale is related to the volume by $(\mathrm{Im}\, r)^3 = \mathrm{e}^{-3\varphi/2}\, \mathcal{V}$, as it follows from eq. (5.13). In this slice of the moduli space the bulk theory can be rewritten in the standard $\mathcal{N}=1$ form employing the chiral fields $s(\theta)$ and $r(\theta)$. The Kähler potential reads

$$K = -\log(\mathrm{Im}\, s) - 3\log(\mathrm{Im}\, r)\,, \tag{A.38}$$

and it coincides with eq. (5.19) re-expressed in the new set of variables. The superpotential is given by

$$W = \frac{1}{\kappa_{10}^2} \int F \wedge \Omega\,, \tag{A.39}$$

where, with respect to eq. (5.20), the O9 projection eliminates the NS-NS flux. It is straightforward to check that

$$\overline{F}^{\bar{s}} = -\mathrm{i}\kappa_4^2\, \mathrm{e}^{K/2} K^{\bar{s}s} D_s W = 8\, \mathrm{e}^{\varphi}\, F_{(0,3)}\,, \tag{A.40}$$

in agreement with eq. (A.36).

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